Name: Solutions ______ / 12

- There are 12 points possible on this proficiency: one point per problem with no partial credit.
- You have 30 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- For at least one problem you must indicate correct use of a constant of integration.
- Circle your final answer.

1. [12 points] Compute the following definite/indefinite integrals.

a. $\int_0^1 \frac{3}{1+x^2}\,dx$

$3\arctan(1) - 3\arctan(0) = 3\frac{\pi}{2} - 3\cdot 0$

$= \frac{3\pi}{2}$

b. $\int e^{3x} - 8x^{\frac{1}{7}} + \sqrt{3}\,dx$

$\frac{1}{3}e^{3x} - \frac{1}{7}x^{8/7} + \sqrt{3}x + C$

c. $\int \frac{x}{x^2-9}\,dx = \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln(|u|) + C$

$u = x^2 - 9$

$du = 2x\,dx$

$= \frac{1}{2}\ln(|x^2-9|) + C$

$= \ln\left(\sqrt{|x^2-9|}\right) + C$

d. $\int (1+\sec(x))^4 \sec(x)\tan(x)\,dx$

$u = 1 + \sec(x)$

$du = \sec(x)\tan(x)\,dx$

$$\int u^4\,du = \frac{u^5}{5} + C = \frac{1}{5}(1+\sec(x))^5 + C$$

e. $\int \frac{\cos(x)}{\sin^3(x)}\,dx = \int u^{-3}\,du = \frac{-1}{2}u^{-2} + C$

$u = \sin(x)$

$du = \cos(x)dx$

$$= -\frac{1}{2}(\sin(x))^{-2} + C$$

$$= \frac{-1}{2\sin^2(x)} + C$$

f. $\int \frac{t^2-2}{\sqrt{t}}\,dt = \int t^{3/2} - 2t^{-1/2}\,dt$

$$= \frac{2}{5}t^{5/2} - 2\,2\,t^{1/2} + C$$

$$= \frac{2}{5}t^{5/2} - 4t^{1/2} + C$$

g. $\int \frac{(1+\ln(x))^2}{x}\,dx$ $= \int u^2\,du = \frac{u^3}{3} + C$

$u = 1 + \ln(x)$ $= \frac{1}{3}(1+\ln(x))^3 + C$

$du = \frac{1}{x}dx$

h. $\int w\sqrt{9-w}\,dw$ $\int (9-u)\sqrt{u}\cdot(-1)\,du$

$u = 9-w$ $= \int (u-9)\sqrt{u}\,du$

$du = -dw$ $= \int u^{3/2} - 9u^{1/2}\,du$

$= \frac{2}{5}u^{5/2} - 9\cdot\frac{2}{3}u^{3/2} + C$

$= \frac{2}{5}(9-w)^{5/2} - 6(9-w)^{3/2} + C$

i. $\int \sin(4x-7)\,dt$

$\frac{-1}{4}\cos(4x-7) + C$

j. $\int e^{2t}\sin\left(e^{2t}\right)\,dt$

$$\int \sin(u)\,du = -\cos(u) + C$$

$u = e^{2t}$

$$= -\cos(e^{2t}) + C$$

$du = e^{2t}\,dt$

k. $\int \frac{1}{(8x-1)^{1/3}}\,dx$

$$\int u^{-1/3}\,\frac{1}{8}\,du = \frac{1}{8}\,\frac{3}{2}\,u^{2/3} + C$$

$u = 8x-1$

$$= \frac{3}{16}(8x-1)^{2/3} + C$$

$du = 8\,dx$

l. $\int t^3 e^{t^4}\,dt$

$$\int e^u\,\frac{1}{4}\,du = \frac{e^u}{4} + C$$

$u = t^4$

$du = 4t^3\,dt$

$$= \frac{1}{4}e^{t^4} + C$$