Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Do not put a "$+C$" where it does not belong and put a "$+C$" in the correct place at least one time.

1. [12 points] Compute the integrals of the following functions.

a. $\int_{-1}^{1}(2x+3)\,dx = x^2+3x\Big]_{-1}^{1} = (1^2+3\cdot 1) - ((-1)^2+3(-1))$

$= 4-(1-3) = 4-(-2) = 6$

b. $\int_{0}^{1} x^2\sqrt{3x^3+1}\,dx = \frac{1}{9}\int_1^4 u^{1/2}\,du = \frac{1}{9}\cdot\frac{2}{3}\cdot u^{3/2}\Big]_1^4$

let $u = 3x^3+1$

$du = 9x^2\,dx$

$\frac{1}{9}du = x^2\,dx$

If $x=0$, $u=1$

$x=1$, $u=4$

$= \frac{2}{27}\left(4^{3/2} - 1^{3/2}\right) = \frac{14}{27}$

c. $\int (\theta + \sin(7\theta))\,d\theta = \frac{1}{2}\theta^2 - \frac{1}{7}\cos(7\theta) + C$

d. $\int 5x^2 e^{x^3}\,dx$ $= \frac{5}{3}\int e^u\,du = \frac{5}{3}e^u + C$

let $u = x^3$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$= \frac{5}{3}e^{x^3} + C$

e. $\int \frac{1}{1+9x^2}\,dx$ $= \int \frac{dx}{1+(3x)^2} = \frac{1}{3}\int \frac{du}{1+u^2} =$

let $u = 3x$

$du = 3\,dx$

$\frac{1}{3}du = dx$

$= \frac{1}{3}\arctan(u) + C$

$= \frac{1}{3}\arctan(3x) + C$

f. $\int (a + be^x + \sec^2(x))\,dx$ $= ax + be^x + \tan(x) + C$

g. $\int \frac{x}{5-3x^2}\,dx$ $= -\frac{1}{6}\int \frac{du}{u} = -\frac{1}{6}\ln|u| + c$

$u = 5-3x^2$

$du = -6x\,dx$

$-\frac{1}{6}du = x\,dx$

$= -\frac{1}{6}\ln|5-3x^2| + c$

h. $\int e^x(1+e^x)^2\,dx$ $= \int u^2\,du = \frac{1}{3}u^3 + c$

$u = 1+e^x$

$du = e^x dx$

$= \frac{1}{3}(1+e^x)^3 + c$

i. $\int \left(\frac{\sqrt{2}}{x} + \frac{3}{x^3} + \frac{\cos(x)}{3}\right) dx$ $= \int \left(\sqrt{2}\cdot x^{-1} + 3x^{-3} + \frac{1}{3}\cos(x)\right) dx$

$= \sqrt{2}\ln|x| - \frac{3}{2}x^{-2} + \frac{1}{3}\sin(x) + c$

j. $\int x(2+x^{1/3})\,dx$ $= \int (2x + x^{4/3})\,dx$

$= x^2 + \frac{3}{7}x^{7/3} + C$

k. $\int 2x^3(1+x^2)^5\,dx$ $= \int x^2(1+x^2)^5\, 2x\,dx = \int (u-1)u^5\,du$

let $u = 1+x^2$

$du = 2x\,dx$

$x^2 = u-1$

$= \int (u^6 - u^5)\,du$

$= \frac{1}{7}u^7 - \frac{1}{6}u^6 + C$

$= \frac{1}{7}(1+x^2)^7 - \frac{1}{6}(1+x^2)^6 + C$

l. $\int (\sec(t)\tan(t)+1)\,dt$

$= \sec(t) + t + C$