**Name:** ______________________________

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 1 hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Do not put a "$+C$" where it does not belong and put a "$+C$" in the correct place at least one time.

**1. [12 points]** Compute the integrals of the following functions.

**a.** $\int_{-1}^{1}(2x+8)\,dx = x^2+8x\Big]_{-1}^{1} = (1^2+8(1)) - ((-1)^2+8(-1))$

$= 9 - (1-8) = 9+7 = 16$

**b.** $\int_{0}^{1} x\sqrt{2x^2+2}\,dx = \frac{1}{9}\int_1^4 u^{1/2}\,du = \frac{1}{9}\cdot\frac{2}{3}\cdot u^{3/2}\Big]_1^4$

let $u=3x^3+1$

$du = 9x^2\,dx$

$\frac{1}{9}\,du = x^2\,dx$

If $x=0$, $u=1$

$x=1$, $u=4$

$= \frac{2}{27}\left(4^{3/2}-1^{3/2}\right) = \frac{14}{27}$

**c.** $\int(\theta+\cos(3\theta))\,d\theta = \frac{1}{2}\theta^2 + \frac{1}{3}\sin(3\theta) + C$

**d.** $\int (n + ke^x + \sec^2(x))\, dx$

$= nx + ke^x + \tan(x) + C$

**e.** $\int 7x^2 e^{x^3}\, dx$ $= \frac{7}{3}\int e^u du$ $= \frac{7}{3} e^u + C$

let $u = x^3$

$du = 3x^2 dx$

$\frac{1}{3} du = x^2 dx$

$= \frac{7}{3} e^{x^3} + C$

**f.** $\int \frac{1}{1+9x^2}\, dx$ $= \int \frac{dx}{1+(3x)^2} = \frac{1}{3}\int \frac{du}{1+u^2} =$

let $u = 3x$

$du = 3dx$

$\frac{1}{3} du = dx$

$= \frac{1}{3}\arctan(u) + C$

$= \frac{1}{3}\arctan(3x) + C$

**g.** $\int \left( \frac{\sqrt{3}}{x} + \frac{3}{x^3} + \frac{\sin(x)}{3} \right) dx$ $= \int \left(\sqrt{3} \cdot x^{-1} + 3x^{-3} + \frac{1}{3}\sin(x)\right) dx$

$= \sqrt{3} \ln|x| - \frac{3}{2}x^{-2} - \frac{1}{3}\cos(x) + C$

**h.** $\int \frac{x}{5-3x^2}\, dx$ $= -\frac{1}{6}\int \frac{du}{u} = -\frac{1}{6}\ln|u| + C$

$u = 5-3x^2$

$du = -6x\, dx$

$-\frac{1}{6}du = x\, dx$

$= -\frac{1}{6}\ln|5-3x^2| + C$

**i.** $\int e^x (1+e^x)^2\, dx$ $= \int u^2\, du = \frac{1}{3}u^3 + C$

$u = 1+e^x$

$du = e^x dx$

$= \frac{1}{3}(1+e^x)^3 + C$

**j.** $\int (\sec(t)\tan(t)+1)\,dt$

$$= \sec(t) + t + C$$

**k.** $\int x(2+x^{2/3})\,dx = \int (2x + x^{5/3})\,dx$

$$= x^2 + \frac{3}{8}x^{8/3} + C$$

**l.** $\int 2x^3(1+x^2)^5\,dx = \int x^2 (1+x^2)^5\, 2x\,dx = \int (u-1)u^5\,du$

let $u = 1+x^2$

$du = 2x\,dx$

$x^2 = u-1$

$$= \int (u^6 - u^5)\,du$$

$$= \frac{1}{7}u^7 - \frac{1}{6}u^6 + C$$

$$= \frac{1}{7}(1+x^2)^7 - \frac{1}{6}(1+x^2)^6 + C$$