Name: Solutions

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have one hour to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions.
- Correct parenthesization is required.
- Do not put a $+C$ where it does not belong, and you must include $+C$ where it is needed.
- You must show sufficient work to justify your final expression. A correct answer for a nontrivial computation with no supporting work will be marked as incorrect.

1. [12 points] Compute the following integrals.

a. $\displaystyle\int_1^4 e^4 + x^{-3} + x^2\,dx \;=\; e^4 x - \frac{1}{2}x^{-2} + \frac{1}{3}x^3 \Big|_1^4$

$= \left(4e^4 - \frac{1}{2}\cdot\frac{1}{16} + \frac{64}{3}\right) - \left(e^4 - \frac{1}{2} + \frac{1}{3}\right)$

$= \dfrac{687}{32} + 3e^4$

b. $\displaystyle\int_0^{1/3} \sin(\pi x)\cos^5(\pi x)\,dx \;=\; \int_1^{\frac{1}{2}} \cancel{\sin(\pi x)}\, u^5 \cdot \frac{du}{-\pi \cancel{\sin(\pi x)}} \;=\; \int_1^{1/2} -\frac{1}{\pi}\, u^5\, du$

$u = \cos(\pi x)$

$\dfrac{du}{dx} = -\pi \sin(\pi x)$

$= -\dfrac{1}{6\pi} u^6 \Big|_1^{1/2} = -\dfrac{1}{6\pi}\left(\frac{1}{2}\right)^6 + \dfrac{1}{6\pi} 1^6$

$= \dfrac{21}{128\pi}$

c. $\displaystyle\int 5e^x + \frac{2}{1+x^2}\,dx \;=\; 5e^x + 2\arctan(x) + C$

d. $\int \frac{3}{2} - \frac{2}{3x}\,dx$ $= \frac{3}{2}x - \frac{2}{3}\ln|x| + C$

e. $\int \sin x + \frac{2}{x^{3/5}}\,dx$ $= \int \sin x + 2x^{-3/5}\,dx = -\cos(x) + 2\cdot\frac{x^{2/5}}{2/5} + C$

$= -\cos(x) + 5x^{2/5} + C$

f. $\int 3e^{2x}\sec^2(e^{2x})\,dx$ $= 3\int \cancel{e^{2x}}\sec^2(u)\cdot\frac{du}{2\cancel{e^{2x}}} = \frac{3}{2}\int \sec^2(u)\,du$

$u = e^{2x}$

$\frac{du}{dx} = 2e^{2x}$ $= \frac{3}{2}\tan(u) + C = \frac{3}{2}\tan\left(e^{2x}\right) + C$

g. $\int 2x\sec(x^2)\tan(x^2)\,dx$ $= \int \cancel{2x} \sec(u)\tan(u)\,\frac{du}{\cancel{2x}} = \sec(u) + C$

$u = x^2$

$\frac{du}{dx} = 2x$

$= \sec(x^2) + C$

h. $\int ax^{-5} + b\sin x\,dx$ $= -\frac{a}{4}x^{-4} - b\cos(x) + C$

$= -\frac{a}{4x^4} - b\cos(x) + C$

i. $\int (\theta-2)(3\theta+1)\,d\theta$ $= \int 3\theta^2 + \theta - 6\theta - 2\,d\theta$

$= \int 3\theta^2 - 5\theta - 2\,d\theta = \theta^3 - \frac{5}{2}\theta^2 - 2\theta + C$

j. $\int 2x(x+3)^{12}\,dx$ $= \int 2(u-3)\,u^{12}\,du = \int 2u^{13} - 6u^{12}\,du$

$u = x+3 \qquad x = u-3$

$\frac{du}{dx} = 1$

$= \frac{1}{7}u^{14} - \frac{6}{13}u^{13} + C$

$= \frac{1}{7}(x+3)^{14} - \frac{6}{13}(x+3)^{13} + C$

k. $\int \frac{1}{x}\frac{1}{\sqrt{1-(\ln x)^2}}\,dx$ $= \int \cancel{\frac{1}{x}} \cdot \frac{1}{\sqrt{1-u^2}}\,\frac{du}{\cancel{1/x}} = \arcsin(u) +$

$u = \ln x$

$\frac{du}{dx} = \frac{1}{x}$

$= \arcsin(\ln(x)) + C$

l. $\int \ln(5) + e^x\,dx$ $= [\ln(5)]\,x + e^x + C$