

Name: _____

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions, but you must show sufficient work to justify your final expression.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}} - \frac{\sqrt{2}}{3} = 2x^{1/2} + 2x^{-1/2} - \frac{\sqrt{2}}{3}$

$$f'(x) = x^{-1/2} - x^{-3/2}$$

$$= \frac{1}{\sqrt{x}} - \frac{1}{x\sqrt{x}}$$

b. $g(x) = \sin(x)\cos(x)$

$$g'(x) = \cos^2(x) - \sin^2(x)$$

c. $h(x) = \frac{\sec(x)}{x}$

$$h'(x) = \frac{\sec(x)\tan(x) \cdot x - \sec(x)}{x^2}$$

$$= \frac{\sec(x)(x\tan x - 1)}{x^2}$$

d. $f(t) = \sin^{-1}(t^2)$

$$f'(x) = \frac{1}{\sqrt{1-t^4}} \cdot 2t$$

e. $f(x) = 3(5^x + 5^2)^3$

$$f'(x) = 9(5^x + 5^2)^2 \cdot \ln(5)5^x$$

f. $f(\theta) = \ln(\theta^3 + \tan(3\theta))$

$$\begin{aligned} f'(\theta) &= \frac{1}{\theta^3 + \tan(3\theta)} \cdot (3\theta^2 + \sec^2(3\theta) \cdot 3) \\ &= 3 \frac{\theta^2 + \sec^2(3\theta)}{\theta^3 + \tan(3\theta)} \end{aligned}$$

g. $y = (x^{0.3} + x)^{-4/3}$

$$y' = -\frac{4}{3} (x^{0.3} + x)^{-7/3} \cdot (0.3x^{-0.7} + 1)$$

h. $f(x) = \csc(\pi/x)$

$$\begin{aligned} f'(x) &= -\csc(\pi/x) \cot(\pi/x) \cdot \left(-\frac{\pi}{x^2}\right) \\ &= \frac{\pi \csc(\pi/x) \cot(\pi/x)}{x^2} \end{aligned}$$

i. $y = e^{-x} + x^4 e^{4x}$

$$y' = -e^{-x} + 4x^3 e^{4x} + x^4 \cdot 4e^{4x}$$

j. $f(x) = \ln\left(\frac{\sin^2(x)}{1-2x}\right)$

$$f'(x) = \frac{1-2x}{\sin^2(x)} \cdot \frac{2 \sin(x) \cos(x) \cdot (1-2x) + 2 \sin^2(x)}{(1-2x)^2}$$

$$= 2 \cot(x) + \frac{2}{1-2x}$$

k. $g(x) = \frac{\cos(2)}{\sqrt[3]{\cos(x)}} = \cos(2) \cdot [\cos(x)]^{-1/3}$

$$g'(x) = -\frac{\cos(2)}{3} \cdot [\cos(x)]^{-4/3} \cdot (-\sin(x))$$

$$= \frac{\cos(2) \sin(x)}{3 (\cos(x))^{4/3}}$$

l. Find $\frac{dy}{dx}$ for $5(x^2 + y^2) = ye^x$. You must solve for $\frac{dy}{dx}$.

$$10x + 10y y' = y' e^x + y e^x$$

$$10x - y e^x = y' e^x - 10y y'$$

$$y' = \frac{dy}{dx} = \frac{10x - y e^x}{e^x - 10y}$$