Name: ______________________________

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions, but you must show sufficient work to justify your final expression.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. **[12 points]** Compute the derivatives of the following functions.

a. $f(x) = x^e + \frac{\pi}{2x} - \frac{4}{\pi^2}$

$f'(x) = e x^{e-1} - \frac{\pi}{2} x^{-2}$

b. $g(x) = x \tan(x)$

$g'(x) = \tan(x) + x \sec^2(x)$

c. $h(x) = \sec^4(3x)$

$h'(x) = 4\sec^3(3x) \cdot \sec(3x) \cdot \tan(3x) \cdot 3$

$= 12 \sec^4(3x) \tan(3x)$

d. $f(x) = \arctan(x^2)$

$f'(x) = \frac{1}{1+x^4} \cdot 2x$

e. $y = (\cos(x) + x^{-3.2})^5$

$y' = 5(\cos(x) + x^{-3.2})^4 \cdot (-\sin(x) - 3.2x^{-4.2})$

f. $f(\theta) = \dfrac{\sqrt{10}}{\sin(\theta)}$ $= \sqrt{10}\csc(\theta)$

$f'(\theta) = -\sqrt{10}\csc\theta \cdot \cot\theta$

g. $y = e^{-x}\cos\left(\frac{x}{3}\right)$

$$y' = -e^{-x}\cos\left(\frac{x}{3}\right) - e^{-x}\sin\left(\frac{x}{3}\right)\cdot\frac{1}{3}$$

h. $y = \ln\left(2\sqrt{x^6 e^x}\right)$

$$y' = \frac{1}{2\sqrt{x^6e^x}}\cdot 2\cdot\frac{1}{2}\left(x^6e^x\right)^{-1/2}\cdot\left(6x^5e^x + x^6e^x\right)$$

$$= \frac{\left(6x^5+x^6\right)e^x}{2x^6e^x} = \frac{6x^5+x^6}{2x^6} = \frac{3}{x} + \frac{1}{2}$$

i. $f(x) = \frac{x^5}{(x^2+2)^3}$

$$f'(x) = \frac{5x^4(x^2+2)^3 - x^5\cdot 3\cdot(x^2+2)^2\cdot 2x}{(x^2+2)^6}$$

$$= \frac{5x^4(x^2+2)^3 - 6x^6\,(x^2+2)^2}{(x^2+2)^6} = \frac{5x^4(x^2+2) - 6x^6}{(x^2+2)^4}$$

j. $f(x) = \cot\left(x^2 - bx\right)$ where b is a constant

$f'(x) = -\csc^2(x^2 - bx) \cdot (2x - b)$

k. $f(x) = \dfrac{x + 2\sin(x)}{\sin(8)}$

$f'(x) = \dfrac{1}{\sin(8)} + \dfrac{2}{\sin(8)} \cos(x)$

l. Find $\dfrac{dy}{dx}$ for $x^2 - y^3 = ye^x$. You must solve for $\dfrac{dy}{dx}$.

$2x - 3y^2 y' = y'e^x + ye^x \rightarrow 2x - ye^x = y'e^x + 3y^2 y'$

$\rightarrow y'(e^x + 3y^2) = 2x - ye^x$

$\rightarrow y' = \dfrac{dy}{dx} = \dfrac{2x - ye^x}{e^x + 3y^2}$