Name: ______________________________

- There are 12 points possible on this proficiency, one point per problem. **No partial credit will be given.**
- You have 60 minutes to complete this proficiency.
- No aids (book, calculator, etc.) are permitted.
- You do **not** need to simplify your expressions, but you must show sufficient work to justify your final expression.
- Your final answers **must start with** $f'(x) =$, $dy/dx =$, or similar.
- **Circle or box your final answer.**

1. [12 points] Compute the derivatives of the following functions.

a. $f(x) = \dfrac{3}{x^{1/3}} + e^{x-1} + \ln(2)$ $= 3x^{-1/3} + e^{x-1} + \ln(2)$

$f'(x) = -x^{-4/3} + e^{x-1} \cdot 1 = e^{x-1} - x^{-4/3}$

b. $g(x) = \dfrac{\cos(x)}{\sin(x)}$ $= \cot(x)$

$g'(x) = -\csc^2(x)$

c. $h(x) = (x - x^5)\sec(x)$

$h'(x) = (1 - 5x^4)\sec(x) + (x - x^5)\sec(x) \cdot \tan(x)$

d. $f(x) = \dfrac{1+e^{-2x}}{\cot(x)}$

$$f'(x) = \frac{(-e^{-2x})\cot(x) + (1+e^{-2x})\csc^2(x)}{\cot^2(x)}$$

e. $f(t) = \dfrac{t\sqrt{t}-\sqrt{10}}{\sqrt{t}}$ $= t - \sqrt{10}\, t^{-1/2}$

$f'(t) = 1 + \dfrac{\sqrt{10}}{2} t^{-3/2}$

f. $f(x) = 2^x \sin(2x)$ $= e^{\ln(2)x} \sin(2x)$

$f'(x) = \ln(2)\, e^{\ln(2)x} \sin(2x) + e^{\ln(2)x} \cos(2x) \cdot 2$

$= \ln(2)\, 2^x \sin(2x) + 2^{x+1} \cos(2x)$

g. $y = x^a \ln(ax+3)$ where a is a constant

$$y' = a x^{a-1} \ln(ax+3) + x^a \cdot \frac{1}{ax+3} \cdot a$$

$$= a x^{a-1} \left[\ln(ax+3) + \frac{x}{ax+3} \right]$$

h. $f(x) = \dfrac{1}{4x} + \left(\dfrac{\pi(x-1)}{3}\right)^2$ $= \frac{1}{4} x^{-1} + \frac{\pi^2}{9}(x-1)^2$

$$f'(x) = -\frac{1}{4} x^{-2} + \frac{\pi^2}{9} \cdot 2(x-1) \cdot 1$$

$$= -\frac{1}{4x^2} + \frac{2\pi^2}{9}(x-1)$$

i. $f(x) = \ln(x + \csc^2(x))$

$$f'(x) = \frac{1}{x+\csc^2(x)} \cdot \left(1 + 2\csc(x) \cdot (-\csc(x)\cot(x)) \right)$$

$$= \frac{1 - 2\csc^2(x)\cot(x)}{x + \csc^2(x)}$$

j. $f(x) = \cos\left(\frac{x}{e^x}\right)$

$$f'(x) = -\sin\left(\frac{x}{e^x}\right) \cdot \left[\frac{e^x - xe^x}{e^{2x}}\right]$$

$$= -\sin\left(\frac{x}{e^x}\right)\left[e^{-x} - xe^{-x}\right]$$

$$= \sin\left(\frac{x}{e^x}\right)(x-1)e^{-x}$$

k. $f(x) = \arcsin\left(\frac{1}{x}\right)$

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{1}{x}\right)^2}} \cdot \left(-1x^{-2}\right)$$

$$= \frac{-1}{x^2\sqrt{1-\left(\frac{1}{x}\right)^2}} = \frac{-1}{\sqrt{x^4 - x^2}} = \frac{-1}{|x|\sqrt{x^2-1}}$$

l. Find $\frac{dy}{dx}$ for $\cos(x) + \sin(y) = xy$. You must solve for $\frac{dy}{dx}$.

$$-\sin(x) + \cos(y)\, y' = y + xy'$$

$$\cos(y) \cdot y' - xy' = y + \sin(x)$$

$$y' = \frac{dy}{dx} = \frac{y + \sin(x)}{\cos(y) - x}$$