

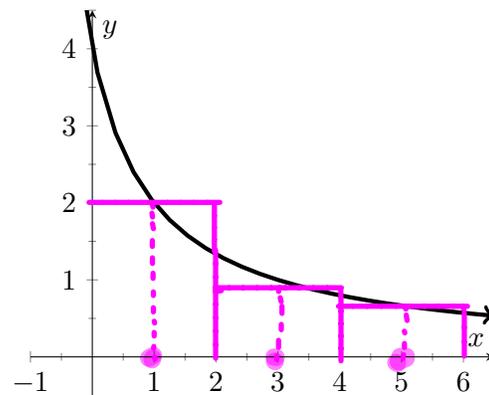
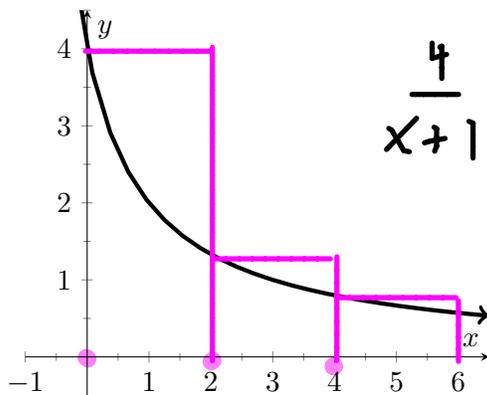
Name: _____ **Solutions** _____

There are 25 points possible on this quiz. This is a closed book quiz. Calculators and notes are not allowed. **Please show all of your work!** If you have any questions, please raise your hand.

Exercise 1. (9 pts.) Estimate the area under $f(x) = \frac{4}{x+1}$ from $x = 0$ to $x = 6$ using three approximating rectangles and

(a.) left endpoints. Sketch the rectangles on the graph below.

(b.) midpoints as sample points. Sketch the rectangles on the graph below.



$$\begin{aligned}
 L_3 &= 2 \left(f(0) + f(2) + f(4) \right) \\
 &= 2 \left(\frac{4}{0+1} + \frac{4}{2+1} + \frac{4}{4+1} \right) \\
 &= 8 \left(1 + \frac{1}{3} + \frac{1}{5} \right) = 8 \left(\frac{15+5+3}{15} \right) \\
 &= \frac{8(23)}{15} = \frac{184}{15}
 \end{aligned}$$

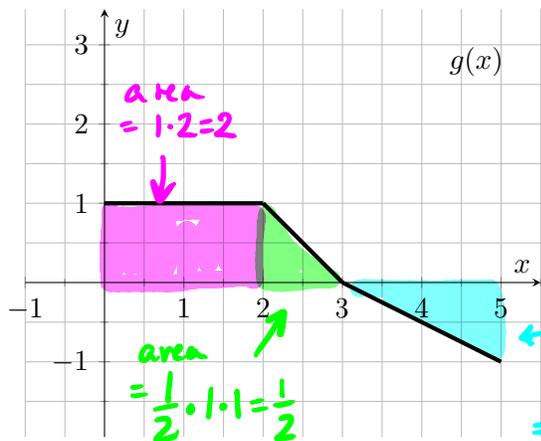
$$\begin{aligned}
 M_3 &= 2 \left(f(1) + f(3) + f(5) \right) \\
 &= 2 \left(\frac{4}{1+1} + \frac{4}{3+1} + \frac{4}{5+1} \right) \\
 &= 2 \left(2 + 1 + \frac{2}{3} \right) = 2 \left(\frac{11}{3} \right) = \frac{22}{3}
 \end{aligned}$$

Exercise 2. (3 pts.) The speed of a skier increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find a lower estimate for the distance she traveled during the first three seconds. Include units with your answer.

time (in seconds)	0	0.5	1	1.5	2	2.5	3
velocity (in feet/sec)	0	4	10	16	20	22	24

$$\begin{aligned}
 \text{distance} &= \frac{1}{2} (0+4+10+16+20+22) = 2+5+16+10+11 \\
 &= 44 \text{ feet}
 \end{aligned}$$

Exercise 3. (4 pts.) Use the graph of $g(x)$ to evaluate the integral $\int_0^5 g(x) dx$.

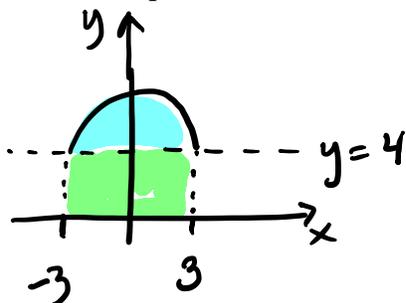


$$\int_0^5 g(x) dx = \text{area above} - \text{area below}$$

$$= 2 + \frac{1}{2} - 1 = 1\frac{1}{2}$$

$$= \frac{3}{2}$$

Exercise 4. (4 pts.) Evaluate the integral $\int_{-3}^3 (\sqrt{9-x^2} + 4) dx$ by interpreting it in terms of areas.



$$= \frac{1}{2} \pi 3^2 + 4 \cdot 6$$

$$= \frac{9}{2} \pi + 24$$

Exercise 5. (5 pts.) Assume that $\int_1^5 f(x) dx = 7$. Use this fact and the properties of integrals to evaluate the integrals below.

(a.) $\int_5^1 f(x) dx = -7$

(b.) $\int_1^5 (3 - 2\pi f(x)) dx$

$$= \int_1^5 3 - 2\pi \int_1^5 f(x) dx$$

$$= 3(5-1) - 2\pi(7)$$

$$= 12 - 14\pi$$