

Name: _____ / 25

Instructor: Bueler | Jurkowski | Maxwell

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [16 points] Compute the derivatives of the following functions. You need not simplify your answers.

a. $r(\theta) = \theta \sec(\theta) \tan(\theta)$

$$r'(\theta) = \sec(\theta) \tan(\theta) + \theta \sec^2(\theta) \tan^2(\theta) + \theta \sec^3(\theta)$$

b. $g(t) = e^{t^2} \sec(t)$

$$g'(t) = 2te^{t^2} \sec(t) + e^{t^2} \sec(t) \tan(t)$$

c. $f(x) = \frac{x^2}{\sqrt{2x+3}}$

$$\begin{aligned} f'(x) &= \frac{2x\sqrt{2x+3} - x^2 \cdot \frac{1}{\sqrt{2x+3}}}{2x+3} = \frac{2x(2x+3) - x^2}{(2x+3)^{3/2}} \\ &= \frac{3x^2 + 3x}{(2x+3)^{3/2}} \end{aligned}$$

d. $s(t) = \tan(e^{\sin(t)})$

$$s'(t) = \sec^2(e^{\sin(t)}) \cdot e^{\sin(t)} \cdot (\cos(t))$$

2. [5 points]

- a. Find the first four derivatives of $y = \cos(4x)$.

$$y' = -4 \sin(4x)$$

$$y'' = -4^2 \cos(4x)$$

$$y''' = 4^3 \sin(4x)$$

$$y'''' = 4^4 \cos(4x)$$

- b. Using part (a), determine the 49th derivative of $y = \cos(4x)$.

$$49 = 4 \cdot 12 + 1$$

$$y^{(49)} = \frac{d}{dx} 4^{48} \cos(4x) = -4^{49} \sin(4x)$$

3. [4 points] Consider the function $f(t) = t - \cos t$.

- a. Find all t values for which $f(t)$ has a horizontal tangent line.

$$f'(t) = 1 + \sin t$$

$$f'(t) = 0 \Rightarrow \sin t = -1$$

$$t = \frac{3\pi}{2} + 2\pi k, k \text{ an integer}$$

- b. Suppose $f(t)$ represents the position in feet of some particle at time t seconds. Find the velocity of the particle at time $t = \frac{\pi}{2}$.

$$f'(\pi/2) = 1 + \sin(\pi/2) = 2 \text{ ft/s}$$