Math 251: Quiz 10 Solutions **Due: Monday Dec 2, 2019**

Name: Solutions ____________ / 25

25 points possible. You may use other sources (books, calculator, friends, etc) but your write up must be your own. The complete quiz is due Monday Dec 2, 2019 at the beginning of class.

1. **[6 points]** Use Part I of the Fundamental Theorem of Calculus to find the derivative of the function.

a. $g(x) = \int_2^x e^t \cos(t^2)dt$

b. $A(x) = \int_{-1}^{x^3} t\ln(3+t^2)dt$

$$\boxed{g'(x) = e^x \cos(x^2)}$$

$$A'(x) = x^3 \ln(3+x^6)(3x^2) = \underline{\underline{3x^5 \ln(3+x^6)}}$$

2. **[9 points]** Evaluate each integral below.

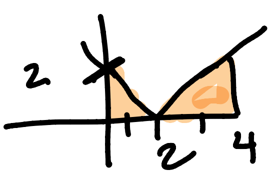
a. $\int_0^3 (ax^2 - bx + 3c)dx = \frac{a}{3}x^3 - \frac{b}{2}x^2 + 3cx \Big]_0^3 = \frac{a}{3}\cdot 3^3 - \frac{b}{2}(3^2) + 3c(3) - (0)$

$$= 9a - \frac{9b}{2} + 9c = 9\left(a - \frac{b}{2} + c\right)$$

b. $\int_0^{\pi/6} \sin(\theta)d\theta = -\cos(\theta)\Big|_0^{\pi/6} = -\cos(\pi/6) - (-\cos 0)$

$$= -\frac{\sqrt{3}}{2} + 1 = 1 - \frac{\sqrt{3}}{2} = \frac{2-\sqrt{3}}{2}$$

c. $\int_0^4 |x-2|dx$ (Hint: Interpret as area, sketch a picture, and compute.)



$$\int_0^4 |x-2|\,dx = 2$$

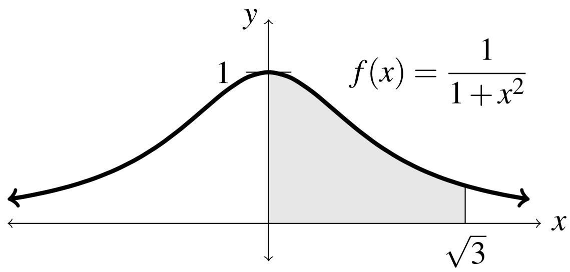
3. **[4 points]** Let $v(t)$ be the velocity (in meters per second) of a particle moving along a line starting at $t = 0$.

a. What does $\int_1^4 v(t)dt$ represent? The displacement of the particle from $t=1$ to $t=4$. OR, The net change in position from $t=1$ to $t=4$.

b. Is it possible for $\int_1^4 v(t)dt$ to be negative? Justify your answer.

Yes. If the position of particle at $t=4$ is behind the position at $t=1$, the displacement would be negative.

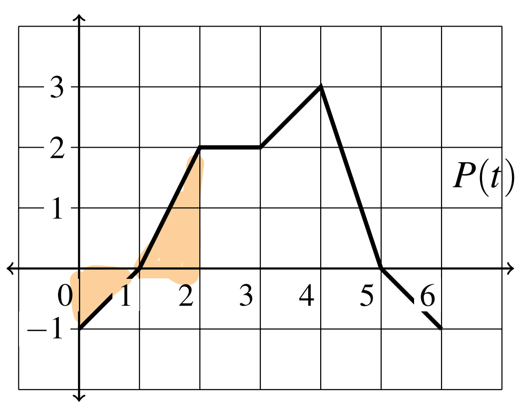
4. [3 points] Find the **exact** value of the area shaded below. Show your work.



$$A = \int_0^{\sqrt{3}} \frac{1}{1+x^2}\,dx = \arctan x \Big|_0^{\sqrt{3}} = \arctan \sqrt{3} - \arctan 0$$

$$= \frac{\pi}{3} - 0 = \frac{\pi}{3}$$

5. [3 points] Let $Q(x) = \int_0^x P(t)dt$, where $P(t)$ is the function whose graph is shown below.



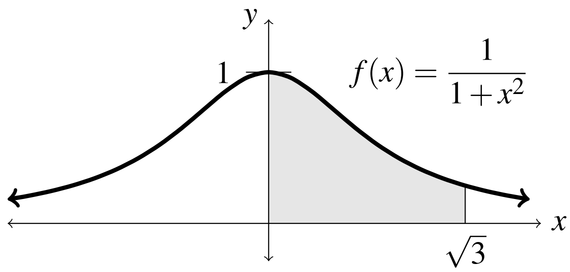
a. Find $Q(2)$ $= -\frac{1}{2} + 1 = \frac{1}{2}$

b. On what interval is $Q(x)$ increasing? $(1, 5)$ (that is, where $P(t) > 0$)

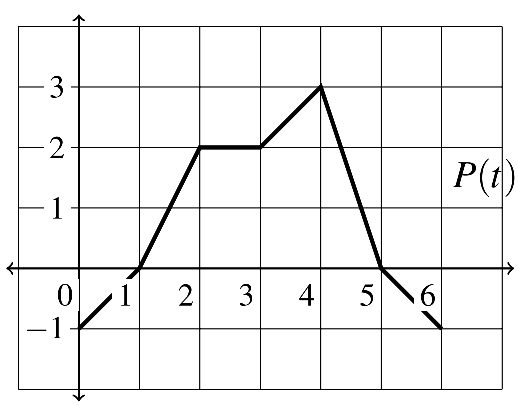
c. Where does $Q(x)$ have a maximum?

at $x=5$. (that is, where $P(t)$ switches from positive to negative.)

4. [3 points] Find the **exact** value of the area shaded below. Show your work.



5. [3 points] Let $Q(x) = \int_0^x P(t)dt$, where $P(t)$ is the function whose graph is shown below.



a. Find $Q(2)$

b. On what interval is $Q(x)$ increasing?

c. Where does $Q(x)$ have a maximum?