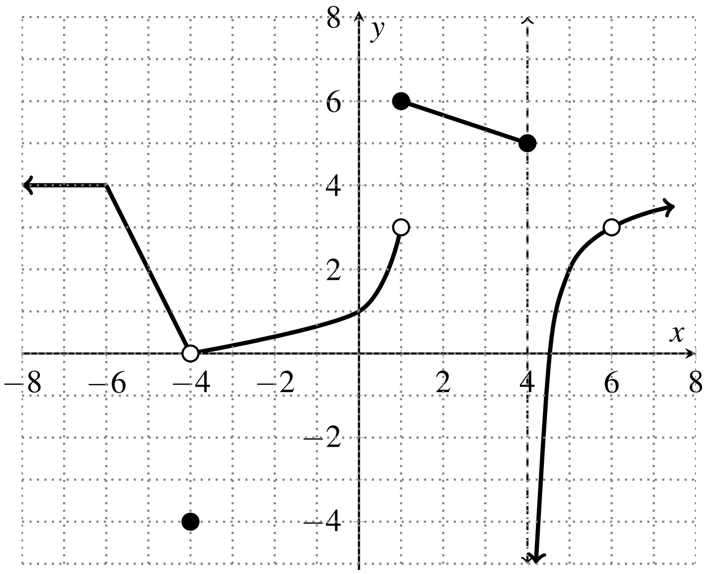
Name: Solutions ______ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [2 points] Use the graph of the function of $f(x)$ to find **all** x-values where $f(x)$ fails to be continuous.



Answer: $x =$ $-4, 1, 4, 6$

2. [4 points]

a. What is wrong with the following equation? $\dfrac{x-4x^3}{x} = 1-4x^2$

It is false when $x=0$ because the left is undefined and the right is 1.

b. In view of part a, explain why the following equation is correct. $\displaystyle\lim_{x\to 0} \frac{x-4x^3}{x} = \lim_{x\to 0} 1-x^2$

Because the limit does not care what happens right at $x=0$. The functions are the same for all other values.

3. [4 points] Explain why the function $f(x) = \begin{cases} 4\sin x & x<0 \\ 0 & x=0 \\ 4x-2 & x>0. \end{cases}$ fails to be continuous at $x=0$.

$\displaystyle\lim_{x\to 0^-} 4\sin x = 0$ but $\displaystyle\lim_{x\to 0^+} 4x-2 = -2$.

So $\displaystyle\lim_{x\to 0} f(x)$ does not exist.

4. [12 points] Evaluate each limit below, if it exists. Show your work to receive full credit. If the limit is infinite, say so; don't just write "DNE".

a. $\lim_{x\to 2} \dfrac{x^2+5x-14}{2+x-x^2}$. $= \lim_{x\to 2} \dfrac{x^2+5x-14}{-(x^2-x-2)} = \lim_{x\to 2} \dfrac{(x-2)(x+7)}{-(x-2)(x+1)}$

$= \lim_{x\to 2} \dfrac{-(x+7)}{x+1} = -\dfrac{9}{3} = -3$

b. $\lim_{h\to 10^-} \dfrac{2|h|-20}{h-10} = \lim_{h\to 10^-} \dfrac{2(|h|-10)}{h-10} = \lim_{h\to 10^-} \dfrac{2(h-10)}{h-10} = \lim_{h\to 10^-} 2 = 2$

b/c $h>0$, so $|h|=h$.

c. $\lim_{x\to 5^+} \left(\dfrac{1}{x-5} - \dfrac{1}{x(x-5)}\right) = \lim_{x\to 5^+} \dfrac{x-1}{x(x-5)} = +\infty$

because as $x\to 5^+$, $x-1>0$ and $x>0$ and $x-5>0$.

Also as $x\to 5^+$, $x-5\to 0^+$.

5. [3 points] What property of the square root function allows you to move the limit inside the square root, as done below.

$$\lim_{x\to 5}\sqrt{x^2+9} = \sqrt{\lim_{x\to 5}(x^2+9)}$$

$f(x)=\sqrt{x}$ is continuous where it is defined