

Name: Solutions

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There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [15 points] Find the derivatives of each of the following. You do not need to simplify your answer.

a. $h(\theta) = e^2 \sec(\theta) + \csc(\theta)$

$$h'(\theta) = e^2 \sec\theta \tan\theta - \cot\theta \csc\theta$$

b. $y = \sin(5x^2)$

$$y' = (\cos(5x^2))(10x)$$

c. $f(x) = \frac{\tan(x)}{x + 4 \cos(x)}$

$$f'(x) = \frac{(x + 4 \cos x)(\sec^2 x) - (\tan x)(1 - 4 \sin x)}{(x + 4 \cos x)^2}$$

d. $f(q) = q^3 e^{6q+5}$

$$f'(q) = (3q^2)(e^{6q+5}) + q^3(e^{6q+5})(6)$$

e. $k(t) = (\sqrt[4]{t} - 7t + 3)^4$

$$k'(t) = 4(t^{1/4} - 7t + 3)^3 \left(\frac{1}{4}t^{-3/4} - 7\right)$$

2. [4 points] Find an x -value such that the function $f(x) = 2x - \sin(4x)$ has a horizontal tangent line. (You do not have to find every value. Simply find one.)

$$f'(x) = 2 - 4\cos(4x) = 0$$

$$4\cos(4x) = 2$$

$$\cos(4x) = \frac{1}{2}$$

We know $\cos\theta = \frac{1}{2}$ when $\theta = \pi/3$.

So we need $4x = \pi/3$.

$$\text{So } x = \pi/12$$

3. [6 points] In a certain experiment involving bacteria, the number N of bacteria in a culture after t days is modeled by the function

$$N(t) = 800 \left(1 + \frac{3}{(t^2 + 1)^2} \right) = 800 \left(1 + 3(t^2 + 1)^{-2} \right)$$

- a. How many bacteria are in the culture at the beginning of the experiment?

beginning means $t=0$.

$$\text{So } N(0) = 800(1+3) = 4 \cdot 800 = 3200 \text{ bacteria}$$

- b. Compute $N'(t)$. (You do not need to simplify, but you may if you choose.)

$$N'(t) = 800(0 + 3(-2)(t^2 + 1)^{-3})(2t) = \frac{-12(800)t}{(t^2 + 1)^3}$$

- c. After one day, is the number of bacteria in the culture **increasing** or **decreasing**, and how do you know? (Justify your answer; an answer with no justification will receive no credit.)

The number of bacteria is decreasing
because $N'(1) < 0$.