

Name: Solutions

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There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [8 points] A small bug is crawling along a branch of a tree. The bug's distance, in millimeters, from the trunk after t seconds is given by the function

$$s(t) = 8t - 3t^2 + \frac{1}{3}t^3.$$

- a. What is the velocity of the bug at time t ?

$$v(t) = s'(t) = 8 - 6t + t^2$$

- b. When is the bug at rest?

Want t when $v=0$. So $0 = 8 - 6t + t^2 = (t-2)(t-4)$

answer: at $t=2$ sec and $t=4$ sec

- c. What is the acceleration of the bug when it's at rest?

$$a(t) = v'(t) = s''(t) = -6 + 2t$$

$$a(2) = -6 + 4 = -2 \text{ mm/s}^2, \quad a(4) = -6 + 8 = 2 \text{ mm/s}^2$$

- d. At time $t = 3$, is the bug moving toward the trunk or away from the trunk? Justify your answer.

$$v(3) = 8 - 18 + 9 = -1 < 0$$

So the bug is moving toward the trunk because $s(t)$ is decreasing

2. [2 points] Let P denote the population of an invasive species of fish that is growing over time, t . Suppose the population P grows at a rate proportional its size. What can you say about the function $P(t)$?

$$\bullet P(t) = P_0 e^{kt}$$

\bullet or $P(t)$ grows exponentially

3. [6points] The edge of a cube was found to be 5 meters with a possible error in measurement of 0.1 meter. Use differentials to estimate the maximum possible error in computing the surface area of the cube. Include units with your answer.

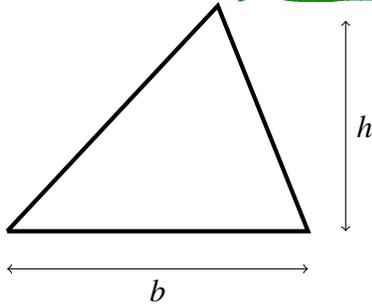
$$s=5 \quad ds = \frac{1}{10}$$

$$\bullet A = 6s^2$$

$$\bullet dA = 6 \cdot 2s \cdot ds$$

$$dA = 12 \cdot 5 \cdot \frac{1}{10} = \boxed{6 \text{ m}^2}$$

4. [9points] The altitude (height, h) of a triangle is increasing at a rate of 2 cm/sec while the area of the triangle is decreasing at a rate of 1 cm²/sec. At what rate is the base, b , of the triangle changing with the altitude is 10 cm and the area is 100 cm²? Give units.



1. Given $\frac{dh}{dt} = 2$ and $\frac{dA}{dt} = -1$

2. Want $\frac{db}{dt}$.

3. Use $A = \frac{1}{2}bh$

aside: When $h=10$
and $A=100$,
 $100 = \frac{1}{2}b \cdot 10$. So $b=20$.

$$4. \quad \frac{dA}{dt} = \frac{1}{2} \left(\frac{db}{dt} h + b \cdot \frac{dh}{dt} \right)$$

$$5. \quad -1 = \frac{1}{2} \left(\frac{db}{dt} \cdot 10 + 20 \cdot 2 \right)$$

$$-2 = 10 \frac{db}{dt} + 40$$

$$\frac{db}{dt} = \frac{-42}{10} = -4.2 \text{ cm/sec}$$