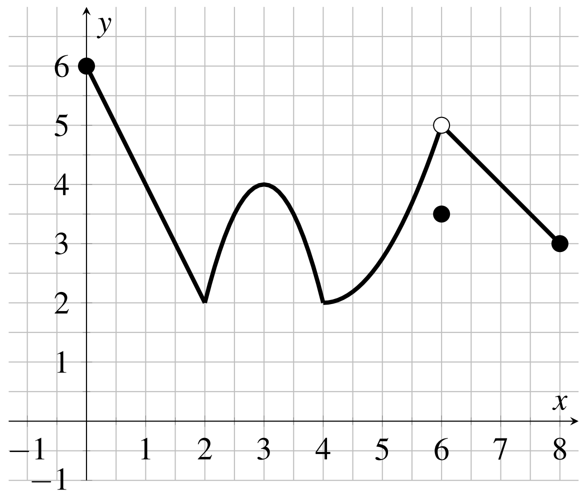
Name: Solutions ____________ / 25

25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. [4 points] Use the graph to determine all the absolute and local maximum and minimum values of the function. If a value does not exist, write DNE.

	y-value	occurs at $x =$
local max (list all)	4	3
local min (list all)	3.5 2	6 2, 4
absolute max	6	0
absolute min	2	2, 4



2. [7 points] Find the absolute maximum and absolute minimum values of

$$f(x) = x^3 + 3x^2 - 9x - 3$$

on the interval $[0,3]$, and the x-values where they occur.

$f'(x) = 3x^2 + 6x - 9$

Solve $f'(x) = 0 \Rightarrow 3x^2 + 6x - 9 = 0 \Rightarrow x^2 + 2x - 3 = 0$

$\Rightarrow (x+3)(x-1) = 0 \Rightarrow x = -3$ or $x = 1$.

only $x=1$ is in the domain. Note $f'(x)$ is defined everywhere.

Test values

x	$f(x)$
0	-3
1	-8
3	24

$f(1) = 1^3 + 3(1)^2 - 9(1) - 3$
$= 1 + 3 - 9 - 3$
$= -8$

$f(3) = 27 + 3(9) - 9(3) - 3$
$= 27 + 27 - 27 - 3$
$= 24$

Absolute Maximum: $y =$ 24 at $x =$ 3

Absolute Minimum: $y =$ -8 at $x =$ 1

3. [8 points]

Consider the function $f(x)$ shown on the graph below, on the interval $[0,2]$. It has the property that $f(0) = 0$ and $f(2) = \frac{3}{2}$.

a. Fill in the blanks: The function $f(x)$ satisfies the hypotheses of the Mean Value Theorem, which means that $f(x)$ is differentiable on $(0,2)$ and continuous on $[0,2]$.

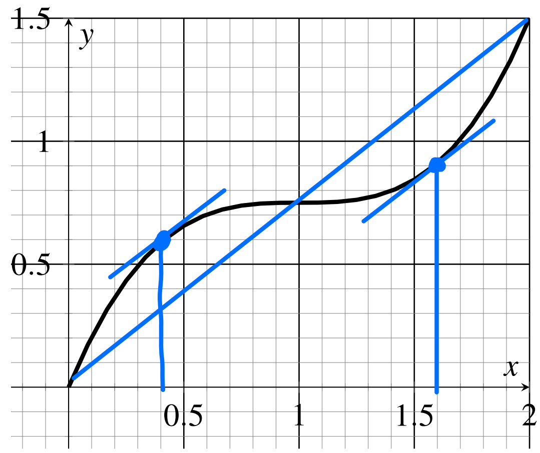
b. What can we conclude about the function $f(x)$, by the Mean Value Theorem? (That is, state the conclusion of the Mean Value Theorem, specified to this function.)

There exists a value $c \in (0,2)$ so that

$$\frac{3/2 - 0}{2 - 0} = f'(c) \Rightarrow$$

$$f'(c) = \frac{3}{2} \cdot \frac{1}{2} = 3/4.$$

c. The graph of $f(x)$ is shown below. $\boxed{\text{Add lines to the graph}}$ to illustrate what the Mean Value Theorem says about this function. Then use the the graph to estimate the value(s) of c whose existence is predicted by the Mean Value Theorem.



Estimated value(s) (to the nearest tenth) of c predicted by MVT (list all): $x = 0.4,\ 1.6$

4. [6 points] Find the critical numbers (critical points) of the function

$$g(x) = \sqrt[3]{x^2 - 9}. \quad = (x^2-9)^{1/3}$$

$$g'(x) = \tfrac{1}{3}(x^2-9)^{-2/3}(2x) = \frac{2x}{3\sqrt[3]{(x^2-9)^2}}$$

$$g'(x) = 0 \Rightarrow x = 0$$

$$g'(x) \text{ DNE} \Rightarrow x^2 - 9 = 0 \Rightarrow x = 3 \text{ or } x = -3$$

Critical points: $x =$ $0, 3, -3.$