Name: Solutions ______ / 20

There are 20 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. **[4 points]** Consider the function $f(x) = \sqrt[3]{4-x}$. Determine all critical points (critical numbers) for $f(x)$.

$x=a$ is a critical point to a function $f(x)$ on its domain if either $f'(a)=0$ or $f'(a)$ DNE.

Domain of $f(x)$ is $\mathbb{R}$

- $f'(x) = -\frac{1}{3}(4-x)^{-2/3} = \dfrac{-1}{3\sqrt[3]{(4-x)^2}} = 0$ never holds since $-1 \neq 0$ $(x \neq 4)$
- $f'(x)$ DNE only when $\boxed{x=4}$ since $\sqrt[3]{(4-x)^2} = 0 \Rightarrow x=4$.

Hence, the only one critical point is $\boxed{x=4}$

2. **[6 points]** Find the absolute maximum and minimum values of the function

$$f(x) = 2x^3 + 3x^2 - 12x + 7$$

on the interval $[0,2]$ and the x-values where they occur. Show your work.

1. Critical points: $f'(x)=0$ or $f'(x)$ DNE

$f'(x) = 6x^2 + 6x - 12$

Since $f'(x)$ is a quadratic function, it has critical points only when $f'(x)=0$.

$f'(x)=0 \Rightarrow 6x^2+6x-12=0 \Rightarrow x^2+x-2=0 \Rightarrow (x+2)(x-1)=0$

Hence, we get $x=-2$, $x=1$. However, $x=-2$ is not in $[0,2]$.

$\boxed{x=1}$ is a critical point.

- $f(1) = 2+3-12+7 = \boxed{0}$

2. Evaluation of f at endpoints:

- $f(0) = \boxed{7}$, $f(2) = 2\cdot 8 + 3\cdot 4 - 24 + 7 = \boxed{11}$

Absolute Maximium: $y =$ 11 occuring at $x =$ 2.

Absolute Minimum: $y =$ 0 occuring at $x =$ 1.

3. [4 points] Consider the function $g(t) = t^2 \ln(t)$.

a. What is the domain of $g(t)$?

$g(t) = g_1(t) \cdot g_2(t)$, where $g_1(t) = t^2$, $g_2(t) = \ln t$

The domain for $g_1(t)$ is $\mathbb{R}$

The domain for $g_2(t)$ is $(0,\infty)$

Hence, g has domain $\boxed{(0,\infty)}$

b. Determine all critical numbers (a.k.a. critical points) of $g(t)$.

Critical points: $g'(t) = 0$ or $g'(t)$ DNE

$g'(t) = 2t\ln t + t$ $\quad$ $g'(t)$ is defined for all t in $(0,\infty)$

- $g'(t) = 0 \Rightarrow 2t\ln t + t = 0 \Rightarrow t(2\ln t + 1) = 0 \Rightarrow t = 0$ or $t = e^{-\frac{1}{2}}$

Since $t=0$ is not in $(0,\infty)$, the only one CP is $\boxed{t = e^{-\frac{1}{2}}}$

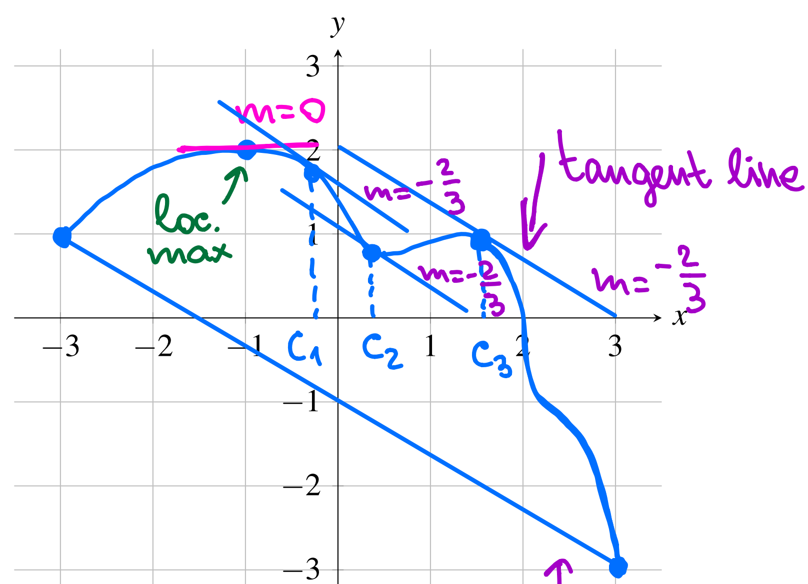
4. [6 points] Suppose h is continuous on $[-3,3]$ and has a derivative at each point in $(-3,3)$, and furthermore, suppose that $h(-3) = 1$ and $h(3) = -3$.

a. What specifically does the Mean Value Theorem let you conclude?

- h is continuous on $[-3,3]$
- h is differentiable on $(-3,3)$

Then there exists c in $(-3,3)$ such that $h'(c) = \dfrac{h(3)-h(-3)}{6} = \dfrac{-4}{6} = \dfrac{-2}{3}$

b. If in addition, you know that h has a local maximum at $x = -1$, draw a diagram that illustrates the Mean Value Theorem for this problem. Your illustration should include a tangent line somewhere.



secant line with the slope $m = -\frac{2}{3}$