

Name: Solutions

/ 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [2 points] Find $\frac{d}{dx}[f(g(h(x)))] = f'(g(h(x))) \cdot g'(h(x)) \cdot h'(x)$

2. [3 points] Use **logarithmic differentiation** to find the derivative of $y = \left(\frac{x^2+1}{\sin(x)+1}\right)^5$

① $\ln y = \ln \left[\left(\frac{x^2+1}{\sin(x)+1}\right)^5 \right] = 5 \left(\ln(x^2+1) - \ln(\sin(x)+1) \right)$

① take "ln" of both sides and simplify

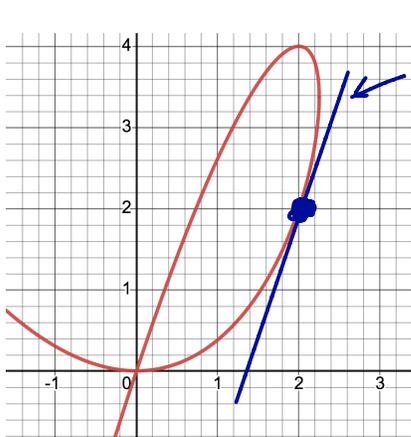
② $\frac{1}{y} \frac{dy}{dx} = 5 \left[\frac{2x}{x^2+1} - \frac{\cos x}{\sin(x)+1} \right]$

② Take derivative implicitly.

③ $\frac{dy}{dx} = y \left(5 \left(\frac{2x}{x^2+1} - \frac{\cos x}{\sin(x)+1} \right) \right) = 5 \left(\frac{x^2+1}{\sin(x)+1} \right)^5 \left(\frac{2x}{x^2+1} - \frac{\cos(x)}{\sin(x)+1} \right)$

③ Solve for $\frac{dy}{dx}$ & resubstitute for y .

3. [5 points] The graph of the equation $x^2 + y^2 = 3xy$ is drawn below. Write an equation of the line tangent to the curve at the point (2,2) and sketch the tangent line on the graph.



$x^2 + y^2 = 3xy$
 $3x^2 + 2y y' = 3xy' + 3y$
 Plug in $x=y=2$:
 $12 + 4y' = 6y' + 6, 6 = 2y', 3 = y'$
 line:
 $y - 2 = 3(x - 2)$ or $y = 2 + 3(x - 2)$
 or $y = 3x - 4$

4. [15 points] Find $\frac{dy}{dx}$ for each expression below.

a. $y = 10 \arctan(2x)$

$$y' = 10 \left(\frac{1}{1+(2x)^2} \right) (2) = \frac{20}{1+4x^2}$$

b. $y = x \sin^{-1}(x)$

$$y' = 1 \cdot \sin^{-1}(x) + x \cdot \frac{1}{\sqrt{1-x^2}} = \sin^{-1}(x) + \frac{x}{\sqrt{1-x^2}}$$

c. $y = \ln(2x+1)$

$$y' = \frac{2}{2x+1}$$

d. $y = e^{-x} + 2e^{x^2} + 3e^2$

$$\begin{aligned} y' &= -e^{-x} + 4xe^{x^2} + 0 \\ &= -e^{-x} + 4xe^{x^2} \end{aligned}$$

e. $y = e^{\ln(x)} = x$

$$y' = 1$$