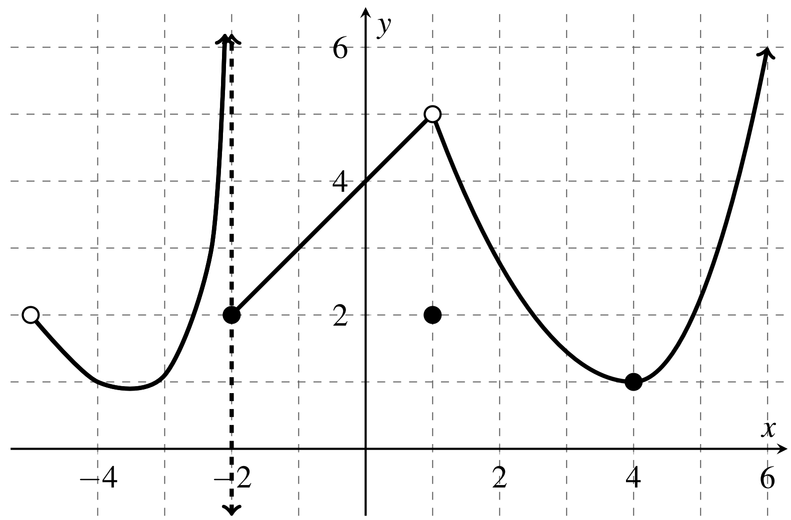
Name: Solutions ______ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (10 points) The function $H(x)$ has domain $(-5,\infty)$ and has a vertical asymptote at $x=-2$. Use the graph of $H(x)$ to answer each question below. If the limit is infinite, indicate that with $\infty$ or $-\infty$. If the value does not exist or is undefined, write **DNE**.

$H(x)$




(a) $H(1) =$ 2 (b) $\lim_{x\to 1} H(x) =$ 5 (c) $\lim_{x\to -2^+} H(x) =$ 2

(d) $H(-2) =$ 2 (e) $\lim_{x\to -2^-} H(x) =$ $\infty$ (f) $\lim_{x\to -2} H(x) =$ DNE

(g) Estimate $H(3)$. 1.5

(h) Evaluate $\lim_{x\to 0}(3H(x)+5)$. $= 3(\lim_{x\to 0} H(x)) + 5 = 3(4)+5 = 17$

(i) List all $x$-values in the domain of $H(x)$ for which the function $H(x)$ fails to be continuous.

$x=-2, \; x=1$

2. (2 points) If $\lim_{x\to -2} f(x) = 6$ and $\lim_{x\to -2} g(x) = -1$, is it possible to evaluate $\lim_{x\to -2} \frac{f(x)+g(x)}{x^2 f(x)}$? If so evaluate the limit. If not, explain why.

$$\lim_{x\to -2} \frac{f(x)+g(x)}{x^2 f(x)} = \frac{6+(-1)}{(-2)^2(6)} = \frac{5}{24}$$

3. (9 points) Use algebra to evaluate the limits below. You must show your work to earn full credit **and** your work will be graded. (That is, you need to write your mathematics correctly.)

   (a) $\lim_{x\to4} \frac{x^2-11x+28}{(x-4)(x+2)} =$ $\lim_{x\to4} \frac{(x-7)(x-4)}{(x-4)(x+2)} = \lim_{x\to4} \frac{x-7}{x+2}$

   $= \frac{4-7}{4+2} = \frac{-3}{6} = -\frac{1}{2}$

   (b) $\lim_{h\to0} \frac{\frac{3}{(a+h)}-\frac{3}{a}}{h} =$ $\lim_{h\to0} \frac{1}{h}\left(\frac{3a}{(a+h)a} - \frac{3(a+h)}{(a+h)(a)}\right)$

   $= \lim_{h\to0} \frac{1}{h}\left(\frac{3a-3a-3h}{a(a+h)}\right) = \lim_{h\to0} \frac{-3h}{h(a)(a+h)} = \lim_{h\to0} \frac{-3}{a(a+h)}$

   $= \frac{-3}{a^2}$

   (c) $\lim_{x\to2} \frac{(x+2)(x-3)}{x^2+4} =$ $\frac{(4)(-1)}{8} = \frac{-1}{2}$

4. (4 points) Let $f(x) = \begin{cases} 1-x+x^2 & x\le 0 \\ e^x & x>0 \end{cases}$.

   (a) Find $\lim_{x\to0^-} f(x)$.

   $\lim_{x\to0^-} f(x) = \lim_{x\to0^-} 1-x+x^2 = 1$

   (b) Find $\lim_{x\to0^+} f(x)$.

   $\lim_{x\to0^+} f(x) = \lim_{x\to0^+} e^x = e^0 = 1$

   (c) Find $f(0)$.

   $f(0) = 1$

   (d) Use your answers to the previous parts to explain whether $f(x)$ is or is not continuous at $x=0$. **Your answer should be a complete sentence.**

$f(x)$ is continuous at $x=0$ because $\lim_{x\to0^-} f(x) = \lim_{x\to0^+} f(x) = f(0) = 1$.