

Name: _____ / 25

There are 18 questions worth 25 points on this quiz. No aids (book, calculator, etc.) are permitted.
Show all work for full credit.

1. [1 points] Determine the domain and range of $f(x) = \frac{1}{x^2} + 5$. Write your answers in interval notation.

Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $[5, \infty)$

2. [1 points] For $f(x) = 8 - x^2$ and $g(x) = 3 + x$, find the composition $f \circ g$ and simplify your answer.

$$\begin{aligned} f(g(x)) &= f(3+x) = 8 - (3+x)^2 \\ &= 8 - (9 + 6x + x^2) \\ &= -1 + 6x + x^2 \end{aligned}$$

$$f \circ g(x) = -1 + 6x + x^2$$

3. [1 points] Write the expression $\frac{x^5 y^8}{x^3 y^{-1} z^2}$ in the form $x^a y^b z^c$. (That is, write the expression with all terms in the numerator.)

$$\frac{x^5 y^8}{x^3 y^{-1} z^2} = x^{5-3} y^{8-(-1)} z^{-2} =$$

$$x^2 y^9 z^{-2}$$

4. [1 points] A rectangle has a width w that is twice its length, ℓ . Find an expression for the area, A , of the rectangle in terms of its length, ℓ .

$$\text{area} = \ell \cdot w = \ell(2\ell)$$

$$A(\ell) = 2\ell$$

5. [2 points] Write an equation of the line between the points $(5, -7)$ and $(2, 1)$.

$$\text{slope} = \frac{1 - (-7)}{2 - 5} = \frac{8}{-3}$$

$$y = -\frac{8}{3}(x - 2) + 1$$

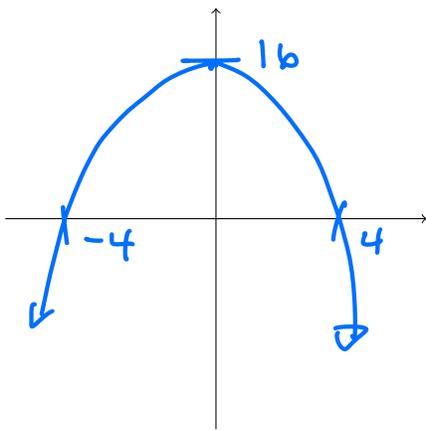
Is the line increasing, decreasing, horizontal or vertical?

decreasing

6. [1 points] Simplify the expression $\frac{2x^3 + 2x^2y}{4x^2 + 12xy}$ by cancelling all common factors in both the numerator and denominator.

$$\frac{2x^3 + 2x^2y}{4x^2 + 12xy} = \frac{2x^2(x+y)}{4x(x+3y)} \quad \hookrightarrow = \frac{x(x+y)}{2(x+3y)} = \frac{x^2 + xy}{2x + 6y}$$

7. [2 points] Sketch the graph of $f(x) = 16 - x^2$. Label any x- or y-intercepts in your sketch.



$$16 - x^2 = 0$$

$$(4-x)(4+x) = 0$$

$$x = 4 \text{ or } x = -4$$

8. [2 points] Use the piecewise defined function $f(x) = \begin{cases} x^3 & x \leq 0 \\ \frac{x}{x+1} & x > 0 \end{cases}$.

a. Find $f(10)$.

$$f(10) = \frac{10}{11}$$

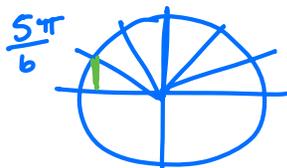
b. Determine x such that $f(x) = -8$.

$$x \leq 0: x^3 = -8 \Rightarrow x = -2$$

$$x > 0: \frac{x}{x+1} = -8 \text{ never true}$$

$$x = -2$$

9. [1 points] Evaluate $\sin(5\pi/6)$ exactly.

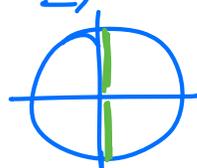


$$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

10. [1 points] Solve the equation $\sin(x) + 1 = 0$ on the interval $0 \leq x < 2\pi$.

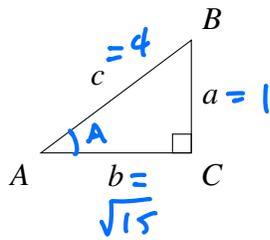
$$\sin(x) + 1 = 0 \Rightarrow$$

$$\sin(x) = -1 \Rightarrow$$



$$x = \frac{3\pi}{2}$$

11. [1 points] In the right triangle below, $a = 1$ and $c = 4$. Determine the value of the tangent function at angle A .

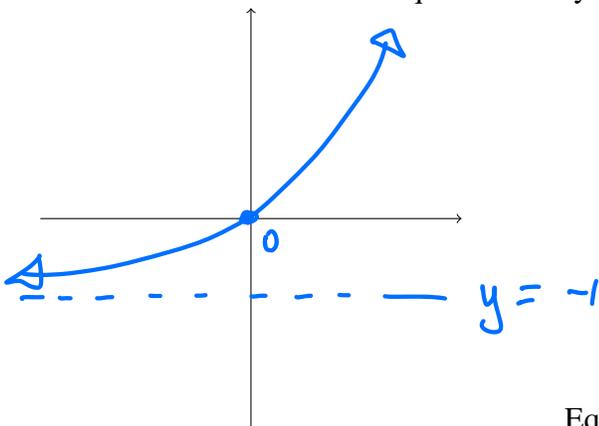


$$4^2 = 1^2 + b^2 \Rightarrow b^2 = 15 \Rightarrow b = \sqrt{15}$$

$$\tan(A) = \frac{\sqrt{15}}{4}$$

$$\tan(A) = \frac{\sqrt{15}}{4}$$

12. [2 points] Sketch the graph of $f(x) = e^x - 1$. Label any x - or y -intercepts, and draw any asymptotes with dashed lines. Give the equation of any asymptotes of $f(x)$.



Equation of asymptote(s)? $y = -1$

13. [1 points] Solve the equation $4 + e^{3x} = 10$. *exactly*.

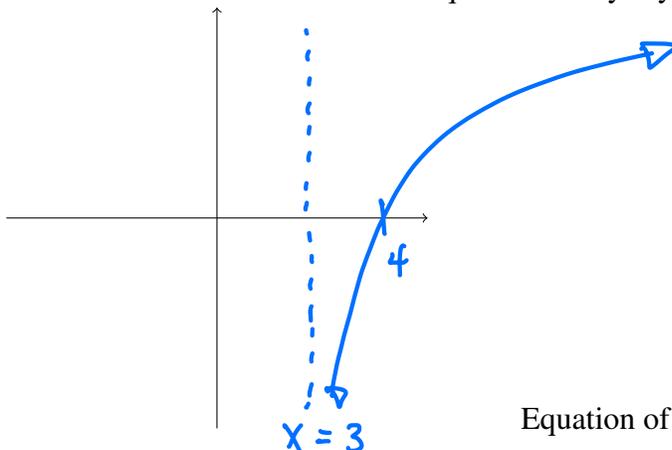
$$4 + e^{3x} = 10 \Rightarrow$$

$$e^{3x} = 6 \Rightarrow$$

$$3x = \ln(6)$$

$$x = \frac{\ln(6)}{3}$$

14. [2 points] Sketch the graph of $f(x) = \ln(x - 3)$. Label any x - or y -intercepts, and draw any asymptotes with dashed lines. Give the equation of any asymptotes of $f(x)$.



Equation of asymptote(s)? $x = 3$

15. [1 points] Solve the equation $\frac{\ln(x+1)}{5} = 3$. *exactly.*

$$\frac{\ln(x+1)}{5} = 3$$

$$\ln(x+1) = 15$$

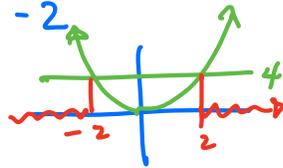
$$x+1 = e^{15}$$

$$x = e^{15} - 1$$

16. [1 points] Solve the inequality $x^2 \geq 4$.

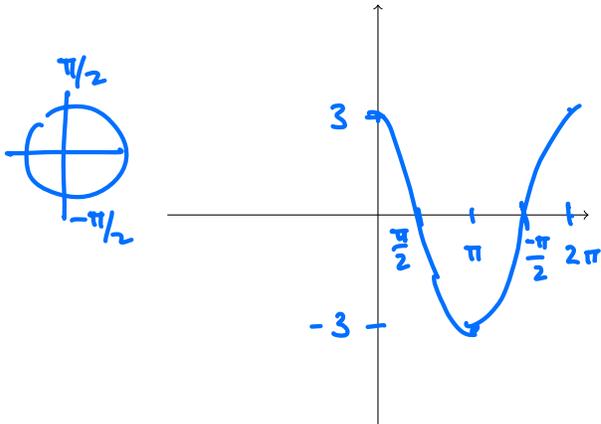
$$x \geq 2 \text{ or } -x \geq 2 \Rightarrow$$

$$x \geq 2 \text{ or } x \leq -2$$



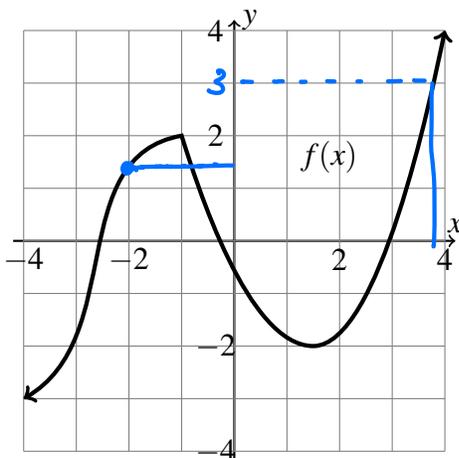
$$x \geq 2 \text{ or } x \leq -2$$

17. [2 points] Sketch the graph of $f(x) = 3 \cos(x)$ on the interval $0 \leq x \leq 2\pi$. Label any x - or y -intercepts. Give the equation of any asymptotes of $f(x)$.



Equation of asymptote(s)? none

18. [2 points] Use the graph of $f(x)$ below to answer the questions.



a. Estimate $f(-2)$. 1.5

b. Estimate an x -value such that $f(x) = 3$. 3.9