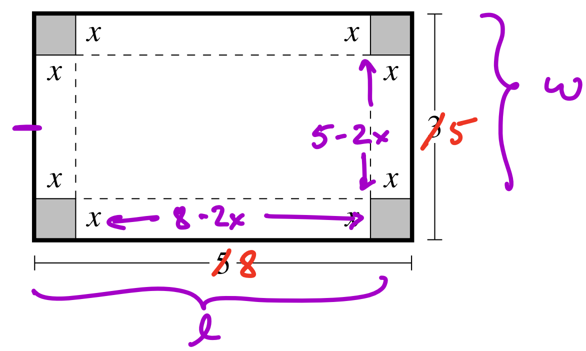
Name: Solutions ________ / 25

There are 25 points possible on this quiz. *You should be able to complete it without using your notes or textbook or a calculator — this is practice for your exams!* If you needed to look something up, you should to me about questions you might have. **Show all work for full credit** and use some words or sentences to help communicate your answers.

1. [8 points] (optimization) You are constructing an **open-topped** cardboard box. You begin with a sheet of cardboard with dimensions ~~3~~ 5 m by ~~5~~ 8 m. You then cut equal-sized squares from each corner so you can fold up the edges to create the sides of the box. What are the dimensions of the box with the largest volume? (See figure to the right. The gray squares have been cut out, and the dashed lines are the fold lines to construct the box.)



a. Identify the quantity to be maximized or minimized.

volume = $l \cdot w \cdot h$ $\quad l = 8-2x,\ w = 5-2x,\ h = x$

b. Write the the quantity to be maximized or minimized as a function of x.

$V(x) = (8-2x)(5-2x)(x)$

c. What is the domain of your function? Use words to explain why that is the domain.

domain = $[0, 5/2]$ since we can't cut out more than half the width of the cardboard

d. Answer the question and use Calculus to demonstrate that you answer is correct. (That is, you need to show that you have found a minimum or maximum.)

$V(x) = (40 - 10x - 16x + 4x^2)x = 40x - 26x^2 + 4x^3$

$V'(x) = 12x^2 - 52x + 40$

$= 4(3x^2 - 13x + 10) = 4(3x-10)(x-1)$

$= 4(3x^2 - 10x - 3x + 10)$ ✓

$\begin{array}{r} 26 \\ 2 \\ \hline 52 \end{array}$

$V'(x) = 0 \Rightarrow x = \frac{10}{3}$ or $x = 1$. Note $x = \frac{10}{3}$ is not in the domain since $\frac{5}{2} = 2.5$ and $\frac{10}{3} = \frac{9}{3} + \frac{1}{3} = 3 + \frac{1}{3}$

Note $V(0) = 0$ and $V(\frac{5}{2}) = 0$. and $V(1) = (8-2)(5-2)(1) = 6(3) = 18$

So $x = 1$ is a max on the domain by the extreme value theorem.

Dimensions of the largest box are: length = 6, width = 3, height = 1

2. [9 points] Evaluate the following limits. You must show your work to earn full credit. If you apply L'Hôpital's Rule, you must indicate this by writing L'H or H above the equals sign where you use the rule.

a. $\lim_{x\to 0} \frac{4e^x - 3x - 12}{4x^3}$

$4 - 0 - 12 = -8$

$0^\pm$

as $x \to 0^+$, $\lim_{x\to 0^+} \frac{4e^x-3x-12}{4x^3} = -\infty$

as $x \to 0^-$ $\lim_{x\to 0^-} \frac{4e^x-3x-12}{4x^3} = \infty$

So the limit itself DNE.

Note L'H does not apply.

b. $\lim_{x\to 0^+} x\ln(x^4)$ type $0\cdot\infty$

$= \lim_{x\to 0^+} \frac{\ln(x^4)}{1/x}$ type $\frac{\infty}{\infty}$ $= \lim_{x\to 0^+} \frac{4\ln(x)}{1/x} \overset{L'H}{=} \lim_{x\to 0^+} \frac{4\cdot \frac{1}{x}}{-1/x^2}$

$= \lim_{x\to 0^+} \frac{-4x^2}{x} = 0$

c. $\lim_{x\to 0} \frac{3x^2 - 2x}{\sin(x)}$ type $\frac{0}{0}$

$\overset{L'H}{=} \lim_{x\to 0} \frac{6x-2}{\cos(x)} = \frac{-2}{1} = -2$

3. [8 points] Evaluate the following indefinite integrals. You must show your work to earn full credit.

a. $\int \left(x^{1/3} + \cos(x) + 5e^x\right) dx$

$= \frac{x^{4/3}}{4/3} + \sin(x) + 5e^x + C$

b. $\int \left(\sec(x)\tan(x) + \frac{x+6}{x}\right) dx$ $= \int \sec(x)\tan(x) + 1 + \frac{6}{x}\, dx$

$= \sec(x) + x + 6\ln|x| + C.$