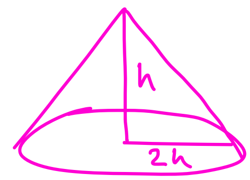
Name: Solutions ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.** You should not be using a calculator on this (or any) quiz.

1. [9 points] Sand is poured onto a surface at a rate of 15 cm^3/sec, forming a conical pile whose base radius is exactly two times its height.

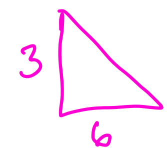
a. Since you know that the base radius is twice the height, write an equation relating r and h. Given that equation, what is the relationship between $\frac{dr}{dt}$ and $\frac{dh}{dt}$?

$$r = 2h \Rightarrow \frac{dr}{dt} = 2\frac{dh}{dt}$$



b. How fast is the height of the pile changing when the pile is 3 cm high? Use the formula $V = \frac{1}{3}\pi r^2 h$ for computing the volume of the cone.

Write a complete sentence to answer the question. Units should be included in your answer.



$$V = \frac{1}{3}\pi r^2 h$$

Method #1 (the easy way)

$$V = \frac{1}{3}\pi (2h)^2 h$$

$$V = \frac{4}{3}\pi h^3$$

$$\frac{dV}{dt} = \frac{4}{3}\pi(3h^2)\frac{dh}{dt}$$

Know $\frac{dV}{dt} = 15$ cm^3/s

want $\frac{dh}{dt}$ when $h = 3$

$$15 = \frac{4}{\cancel{3}}\pi(\cancel{3})(3^2)\frac{dh}{dt} \Rightarrow$$

$$\frac{dh}{dt} = \frac{15}{36\pi} = \frac{5}{12\pi}$$

Method #2 (the hard way)

$$\frac{dV}{dt} = \frac{\pi}{3}\left[2r\frac{dr}{dt}h + \frac{dh}{dt}r^2\right]$$

$$15 = \frac{\pi}{3}\left[2(6)\left(2\frac{dh}{dt}\right)(3) + \frac{dh}{dt}(36)\right]$$

$$15 = 24\pi\frac{dh}{dt} + 12\pi\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{15}{36\pi} = \frac{5}{12\pi}$$

Answer: When the pile is 3cm high, the height is changing (increasing) at a rate of $\frac{5}{12\pi}$ cm/s.

2. [8 points] Consider the function $f(x) = \sqrt{4-x}$.

a. Find the linearization (linear approximation) $L(x)$ of the function $f(x)$ at $a = 0$.

$f'(x) = \frac{1}{2}(4-x)^{-1/2}(-1) = \frac{-1}{2\sqrt{4-x}}$ $f(0)=2$

$f'(0) = \frac{-1}{4}$

$L(x) = -\frac{1}{4}(x-0) + 2$

b. What is x if $f(x) = \sqrt{3.9}$? Give your answer as a fraction. $4-x = 3 - \frac{1}{10}$ so $x = \frac{1}{10}$

c. Use linearization or differentials to **estimate** $\sqrt{3.9}$. Clearly show your work.

$L(\frac{1}{10}) = -\frac{1}{4}(\frac{1}{10}) + 2 = 2 - \frac{1}{40} = \frac{79}{40}$

So $\sqrt{3.9} \approx 2 - \frac{1}{40} = \frac{79}{40}$

3. [8 points] Let $f(x) = (4-x^2)^2$.

a. Find all critical points for $f(x)$. Show your work.

$f'(x) = 2(4-x^2)(-2x)$

$f'(x) = 0 \Rightarrow 4 - x^2 = 0 \Rightarrow \boxed{x=2}$ or $\boxed{x=-2}$ OR $-2x = 0 \Rightarrow \boxed{x=0}$

b. Determine the absolute maximum and absolute minimum of $f(x)$ on the interval $[0,3]$ or state that none exist. You must show your work to receive full credit. See the answer-blank below.

end pt $\rightarrow f(0) = (4-0^2)^2 = 16$

critical pt $\rightarrow f(2) = (4-2^2)^2 = 0$

endpt $\rightarrow f(3) = (4-3^2)^2 = (4-9)^2 = 25$

$f(-2) = (4-4)^2 = 0$ as well but -2 is not in our interval

maximum value of $f(x)$ for x in [0,3]: 25

x**-value(s)** where the maximum value of $f(x)$ occurs: $x = 3$

minimum value of $f(x)$ for x in [0,3]: 0

x**-value(s)** where the minimum value of $f(x)$ occurs: $x = 2$