Name: Solutions ____________ / 25

There are 25 points possible on this quiz. *You should be able to complete it without using your notes or textbook or a calculator — this is practice for your exams!* If you needed to look something up, you should to me about questions you might have. **Show all work for full credit** and use some words or sentences to help communicate your answers.

1. **[12 points]** The following questions concern the function $k(x) = \frac{3}{2}x^4 - 3x^3$ You must **show all your work** and explain your answers. Here are the first and second derivatives of $k(x)$:

$$k'(x) = 6x^3 - 9x^2; \qquad k''(x) = 18x^2 - 18x.$$

a. Identify all critical points of $k(x)$.

$k'(x) = 0 \Rightarrow 6x^3 - 9x^2 = 0 \Rightarrow 3x^2(2x-3) = 0$

$\Rightarrow x = 0$ or $x = +\frac{3}{2}$

b. Determine intervals where $k(x)$ is increasing or decreasing.

x	-1	0	1	$3/2$	2
k'	$-$	0	$-$	0	$+$
k	$\searrow$	$-$	$\searrow$	$-$	$\nearrow$

$k'(-1) = 3(-1)^2(2(-1)-3) = (+)(-)$

$k'(1) = 3(1)^2(2-3) = (+)(-)$

$k'(2) = 3(2)^2(4-3) = (+)(+) = +$

$\boxed{\text{Answer: } k(x) \text{ is decreasing on } (-\infty, 3/2) \text{ and increasing on } (3/2, \infty)}$

c. Identify the location (x-values) of any local maxima or minima of $k(x)$ or state that none exist.

$x = 3/2$ is a local min

there are no local maxima

d. Determine intervals where $k(x)$ is concave up and concave down.

$k''(x) = 0 \Rightarrow 18x^2 - 18x = 0 \Rightarrow 18x(x-1) = 0 \Rightarrow x = 0$ or $x = 1$

x	-1	0	$1/2$	1	2
k''	$+$	0	$-$	0	$+$
k	$\cup$		$\cap$		$\cup$

$k''(-1) = 18(-1)(-2) = +$

$k''(1/2) = 18(1/2)(1/2 - 1) = (+)(-)$

$k''(2) = 18(2)(2-1) = +$

$\boxed{\text{Answer: } K \text{ is concave up on } (-\infty,0)\cup(1,\infty) \text{ and concave down on } (0,1)}$

e. Identify the x-values of any inflection points of $k(x)$ or state that none exist.

$\boxed{x = 0 \text{ \& } x = 1 \text{ are both inflection points.}}$

2. [8 points] Evaluate the limits below. **You must justify your answer algebraically to receive full credit.** (This means: show your work, using calculus skills and techniques.)

a. $\lim_{x \to -\infty} \dfrac{6x^3 - 4x^2 + 5}{10 - 2x - 8x^3}$ $= \lim_{x \to -\infty} \dfrac{6 - 4/x + 5/x^3}{10/x^3 - 2/x^2 - 8} = -\dfrac{6}{8} = -\dfrac{3}{4}$

b. $\lim_{x \to \infty} \dfrac{4x - 2}{\sqrt{5x^2 - 4}}$ $= \lim_{x \to \infty} \dfrac{4x - 2}{\sqrt{x^2(5 - 4/x^2)}} = \lim_{x \to \infty} \dfrac{x(4 - 2/x)}{x\sqrt{5 - 4/x^2}}$ (since $x \to \infty \Rightarrow x > 0$)

$= \lim_{x \to \infty} \dfrac{4 - \underbrace{2/x}_{\to 0}}{\sqrt{5 - \underbrace{4/x^2}_{\to 0}}} = \dfrac{4}{\sqrt{5}}$

3. [5 points] Let $f(x) = \dfrac{2x^2 + 7x + 6}{x^2 - 4x + 4} = \dfrac{(2x+3)(x+2)}{(x-2)^2}$

a. Give the equation of any vertical asymptotes and **justify your answer using limits and the calculus definition of a vertical asymptote.**

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \dfrac{\underbrace{(2x+3)}_{\to 7}\underbrace{(x+2)}_{\to 4}}{\underbrace{(x-2)^2}_{\to 0^+}} = \infty$

Since $f(x) \to \infty$ as $x \to 2^+$, the line $x = 2$ is a vertical asymptote

b. Give the equation of any horizontal asymptotes and **justify your answer using limits and the calculus definition of a horizontal asymptote.**

$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \dfrac{2 + 7/x + 6/x^2}{1 - 4/x + 4/x^2} = 2$

$\lim_{x \to -\infty} f(x) = \lim_{x \to \infty} \dfrac{2(-x)^2 + 7(-x) + 6}{(-x)^2 - 4(-x) + 4} = \lim_{x \to \infty} \dfrac{2x^2 - 7x + 6}{x^2 + 4x + 4}$

$= \lim_{x \to \infty} \dfrac{2 - 7/x + 6/x^2}{1 + 4/x + 4/x^2} = 2.$ So $y = 1$ is a HA (in both directi