Name: Key _____ / 25

Please circle your section: 901 (Gossell) 902 (Gossell) 903 (Graves-McCleary)

No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[12 points]** Find the derivatives of each of the following functions. Use correct derivative notation. You do not need to simplify your answers.

   a. $f(x) = e^x + x^e + e^e$

   $f'(x) = e^x + ex^{e-1}$

   b. $g(x) = \arctan(x^2)$

   $g'(x) = \frac{1}{1+(x^2)^2} \cdot 2x = \frac{2x}{1+x^4}$

   c. $h(x) = \log_3(x)\arccos(e^x)$

   $h'(x) = \frac{1}{x\,\ln(3)} \arccos(e^x) + \log_3(x) \cdot \frac{-1}{\sqrt{1-(e^x)^2}} \cdot e^x$

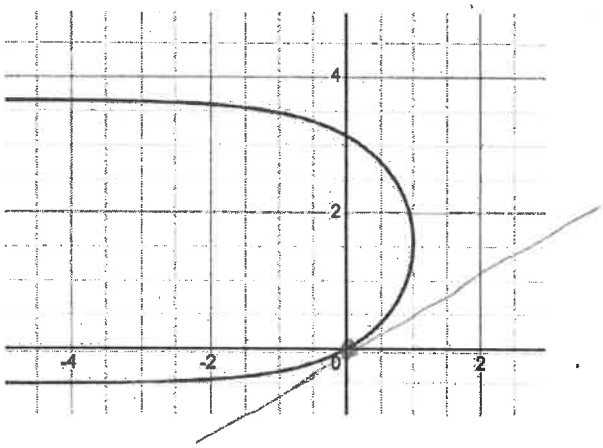
   d. $r(\theta) = \ln\left(\frac{\cos(\theta)}{\sqrt{2\theta+3}}\right)$ (Hint: You may want use log properties to simplify first.)

   $r(\theta) = \ln(\cos(\theta)) - \frac{1}{2}\ln(2\theta+3)$

   $r'(\theta) = \frac{1}{\cos(\theta)}(-\sin(\theta)) - \frac{1}{2} \cdot \frac{1}{2\theta+3} \cdot 2$

2. [9 points] Part of the graph of $3^x - 2\sin(y) = 1$ is shown below.

a. Sketch the **tangent line** to this graph at the point $(0,0)$.



b. Use implicit differentiation to find $\frac{dy}{dx}$.

$$\frac{d}{dx}\left[3^x - 2\sin(y)\right] = \frac{d}{dx}[1]$$

$$3^x \ln(3) - 2\cos(y)\cdot\frac{dy}{dx} = 0$$

$$3^x \ln(3) = 2\cos(y)\cdot\frac{dy}{dx} \quad \Rightarrow \quad \boxed{\frac{dy}{dx} = \frac{3^x \ln(3)}{2\cos(y)}}$$

c. Use your answer in (b) to find the **slope** of the tangent line to $(0,0)$.

$$\left.\frac{dy}{dx}\right|_{(0,0)} = \frac{3^0 \ln(3)}{2\cos(0)} = \boxed{\frac{\ln(3)}{2}}$$

3. [4 points] Considter the function $y = (3x+1)^x$.

a. Write down **your first step** in finding $\frac{dy}{dx}$.

$\boxed{\ln(y) = \ln\left((3x+1)^x\right)}$ OR $\boxed{y = e^{x\ln(3x+1)}}$

b. Choose the correct expression for $\frac{dy}{dx}$. **(Circle one.)**

(I) $3x(3x+1)^{x-1}$

(II) $3x(3x+1)^{x-1}\left(\frac{\ln(3x+1)}{3x+1}\right)$

(III) $(3x+1)^x\left(\ln(3x+1)+\frac{x}{3x+1}\right)$

(IV) $(3x+1)^x\ln(3x+1)$

(V) $(3x+1)^x\left(\ln(3x+1)+\frac{3x}{3x+1}\right)$

(VI) $(3x+1)^x+\frac{\ln(3x+1)}{3x+1}$