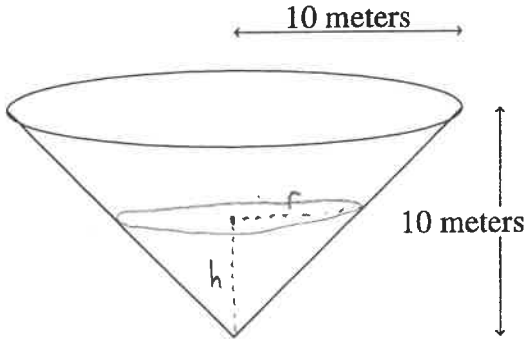
Name: Key _____ / 25

Please circle your section: 901 (Gossell) 902 (Gossell) 903 (Graves-McCleary)

No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[10 points]** A conical tank is being filled with water at a constant rate of 50 cubic meters per minute. The height and radius of the tank are both 10 meters. (See picture below.)



The volume of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$ where V is volume, r is the radius of the top of the cone and h is the height of the cone.

Using this infomation, **determine how fast the height of the water is changing** when the height of the water is 5 meters. Include units with your answer.

Let h be the height of the water and r be radius of the surface of the water.

The ratio $\frac{r}{h}$ should always equal the ratio $\frac{10}{10}$. Therefore, $\frac{r}{h} = \frac{10}{10} \Rightarrow 10r = 10h \Rightarrow r = h$.

So $V = \frac{1}{3}\pi(h^2)h = \frac{1}{3}\pi h^3$ (since $r=h$). We are given that $\frac{dV}{dt} = 50\ \frac{m^3}{min}$ and we want to find $\frac{dh}{dt}$.

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\tfrac{1}{3}\pi h^3\right]$$

$$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$$

Plug in information $\rightarrow$ $50 = \pi(5)^2 \frac{dh}{dt} \Rightarrow \frac{50}{25\pi} = \frac{dh}{dt} \Rightarrow \boxed{\frac{dh}{dt} = \frac{2}{\pi}\ \frac{m}{min}}$

2. [6 points] Let $f(x) = \sin(x) + e^x$ Note that $f'(x) = \cos(x) + e^x$

a. Find the **linear approximation**, $L(x)$, to $y = f(x)$ near $a = 0$.

$L(x) = f(a) + f'(a)(x-a)$ $a=0$, $f(a) = \sin(0) + e^0 = 1$, $f'(a) = \cos(0) + e^0 = 2$

so $\boxed{L(x) = 1 + 2(x-0) \text{ or } L(x) = 1 + 2x}$

b. Use the linear approximation from part (a) above to estimate the value of $\sin(0.5) + e^{0.5}$. (You are trying to approximate $f(0.5)$. **You need to simplify your answer for full credit.**)

$L(0.5) = 1 + 2(0.5) = 1 + 1 = 2$ so $\boxed{f(0.5) \approx 2}$

3. [9 points] Let $f(x) = \dfrac{x}{x^2+1}$.

a. Find $f'(x)$.

$$\boxed{f'(x) = \frac{1(x^2+1) - x(2x)}{(x^2+1)^2}}$$

b. Find **all critical points** of $f(x)$.

$(x^2+1)^2$ is never 0

Set $1(x^2+1) - x(2x) = 0 \Rightarrow x^2 + 1 - 2x^2 = 0 \Rightarrow 1 = x^2 \Rightarrow$ $\boxed{x = 1 \text{ and } x = -1 \text{ are critical numbers}}$

c. Determine the **absolute maximum** and **absolute minimum** y-values of $f(x)$ on the interval $[0, 10]$. Show your work.

-1 is not in $[0,10]$. We'll test 0, 1, and 10.

x	$f(x)$	
0	0	← smallest
1	$\frac{1}{2}$	← largest
10	$\frac{10}{101}$	

$\boxed{\text{Absolute max is } y = \frac{1}{2} \text{ at } x = 1 \quad \text{Absolute min is } y = 0 \text{ at } x = 0}$