Name: _____ _____ / **25**

Please circle your section: 901 (Gossell) 902 (Gossell) 903 (Graves-McCleary)

No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [12 points] Answer the following questions about the function $f(x) = \dfrac{4x}{x^2+1}$.

Note that $f'(x) = \dfrac{4-4x^2}{(x^2+1)^2}$ and $f''(x) = \dfrac{8x^3-24x}{(x^2+1)^3}$.

a. Determine the intervals where $f(x)$ is **increasing** and where $f(x)$ is **decreasing**. Show your work.

Increasing: _____ Decreasing: _____

(Use interval notation. If none write "none".)

b. $f(x)$ has a local maximum at $x =$ _____ and a local minimum at $x =$ _____.

(If none, write "none".)

c. Find all intervals where f is **concave up** and where f is **concave down**. Show your work.

Concave up: _____ Concave down: _____

(Use interval notation. If none write "none".)

d. $f(x)$ has (an) inflection point(s) at $x =$ _____. *(If none, write "none".)*

e. $f(x)$ has a **vertical asymptote** at $x =$ _____ and a **horizontal asymptote** at $y =$ _____.

(If none, write "none".)

2. [4 points] Evalutate $\lim_{x\to\infty} \dfrac{100x^2}{3+2x-x^2}$. Show your work and use proper limit notation.

3. [9 points] Graph a function of a function $h(x)$ that satisfies the following properties:

- The domain of $h(x)$ is $(-\infty,\infty)$.
- $h(0)=2$ and $h(2)=4$
- $\lim_{x\to-\infty} h(x)=0$
- $\lim_{x\to\infty} h(x)=-\infty$
- $h'(x)>0$ when $x<2$
- $h'(x)<0$ when $x>2$
- $h''(x)>0$ when $x<0$
- $h''(x)<0$ when $x>0$

- **Label** any local maximums by writing LOCAL MAX, local minimums by writing LOCAL MIN, inflection points by writing IP
- **Draw** any horizontal or vertical asymptotes with dashed lines and **label** each one with its equation.

