Name: Key _____ / 25

Please circle your section: 901 (Gossell) 902 (Gossell) 903 (Graves-McCleary)

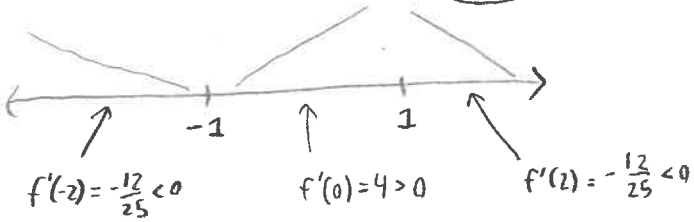
No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[12 points]** Answer the following questions about the function $f(x) = \dfrac{4x}{x^2+1}$.

Note that $f'(x) = \dfrac{4-4x^2}{(x^2+1)^2}$ and $f''(x) = \dfrac{8x^3-24x}{(x^2+1)^3}$.

a. Determine the intervals where $f(x)$ is **increasing** and where $f(x)$ is **decreasing**. Show your work.

Critical numbers: $4-4x^2=0 \Rightarrow x^2=1 \Rightarrow x=\pm 1$. $(x^2+1)^2 \neq 0$



Increasing: $(-1, 1)$ Decreasing: $(-\infty, -1) \cup (1, \infty)$

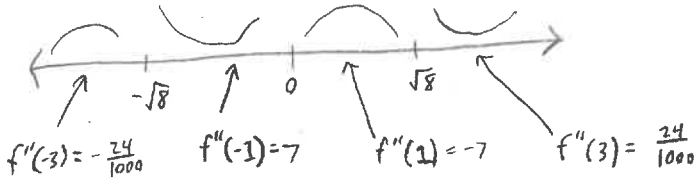
(Use interval notation. If none write "none".)

b. $f(x)$ has a local maximum at $x =$ 1 and a local minimum at $x =$ -1.

(If none, write "none".)

c. Find all intervals where f is **concave up** and where f is **concave down**. Show your work.

$8x^3-24x=0 \Rightarrow 8x(x^2-8)=0 \Rightarrow x=0,\ x=\pm\sqrt{8}$ $(x^2+1)^3 \neq 0$



Concave up: $(-\sqrt{8}, 0) \cup (\sqrt{8}, \infty)$ Concave down: $(-\infty, -\sqrt{8}) \cup (0, \sqrt{8})$

(Use interval notation. If none write "none".)

d. $f(x)$ has (an) inflection point(s) at $x =$ $-\sqrt{8}$, 0, $\sqrt{8}$. *(If none, write "none".)*

e. $f(x)$ has a **vertical asymptote** at $x =$ none and a **horizontal asymptote** at $y =$ 0.

(If none, write "none".)

$\lim_{x\to\pm\infty} \dfrac{4x}{x^2+1} = \lim_{x\to\pm\infty} \dfrac{\frac{4}{x}}{1+\frac{1}{x^2}} = 0$

2. [4 points] Evalutate $\lim_{x\to\infty} \dfrac{100x^2}{3+2x-x^2}$. Show your work and use proper limit notation.

$$= \lim_{x\to\infty} \frac{\frac{100x^2}{x^2}}{\frac{3}{x^2}+\frac{2x}{x^2}-\frac{x^2}{x^2}} = \lim_{x\to\infty} \frac{100}{\frac{3}{x^2}+\frac{2}{x}-1} = \frac{100}{-1} = \boxed{-100}$$

3. [9 points] Graph a function of a function $h(x)$ that satisfies the following properties:

- The domain of $h(x)$ is $(-\infty,\infty)$.
- $h(0)=2$ and $h(2)=4$
- $\lim_{x\to-\infty} h(x)=0$
- $\lim_{x\to\infty} h(x)=-\infty$
- $h'(x)>0$ when $x<2$
- $h'(x)<0$ when $x>2$
- $h''(x)>0$ when $x<0$
- $h''(x)<0$ when $x>0$

- **Label** any local maximums by writing LOCAL MAX, local minimums by writing LOCAL MIN, inflection points by writing IP
- **Draw** any horizontal or vertical asymptotes with dashed lines and **label** each one with its equation.

