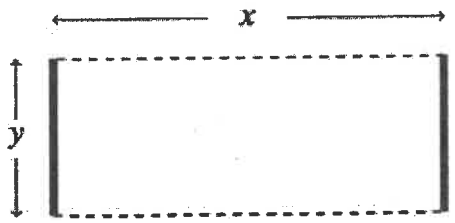
Name: Key _____ / 25

Please circle your section: 901 (Gossell) 902 (Gossell) 903 (Graves-McCleary)

No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[10 points]** You need to construct a rectangular fence that encloses an area of 100 square feet. The two vertical sides (drawn solid below) will be made of material that costs \$12 per foot while the material for the horizontal sides (drawn dashed below) costs \$3 per foot.



a. Express the **cost** of the fence as a function in terms of 1 variable.

$C = 3(2x) + 12(2x)$, $xy = 100 \Rightarrow y = \frac{100}{x}$

$$C(x) = 6x + 24\left(\frac{100}{x}\right) = 6x + 2400x^{-1}$$ (Note: $x > 0$)

b. Use calculus the find the dimensions of the **least expensive** fence. Justify your answer.

$C'(x) = 6 - 2400x^{-2}$

Critical Numbers: $6 - 2400x^{-2} = 0 \Rightarrow 6x^2 - 2400 = 0 \Rightarrow 6(x^2 - 400) = 0$

$\Rightarrow 6(x - 20)(x + 20) = 0$

$x = 20$ $x = -20$ ← not in domain

Test $x = 20$: $C''(x) = 4800x^{-3}$

$C''(20) = 4800(20)^{-3} > 0$

$\Rightarrow x = 20$ is a local minimum

and must be the absolute minimum

since it is the only critical number on the domain.

$x = 20$ $y = \frac{100}{20} = 5$

2. [9 points] Evaluate the following limits. **Show your work**, uncluding appropriate use of limit notation. If you use L'Hôpital's rule, you must indicate where you are using it by writing $\overset{H}{=}$ or $\overset{L'H}{=}$ or something similar. Use $+\infty$ or $-\infty$ where appropriate, and if the limit does not exist, write DNE and provide a justification.

a. $\lim_{x\to 2} \dfrac{x^3-5x^2+9x-6}{6x-3x^2} \overset{L'H}{=} \lim_{x\to 2} \dfrac{3x^2-10x+9}{6-6x} = \dfrac{12-20+9}{6-12} = \dfrac{1}{-6} = \boxed{-\dfrac{1}{6}}$

I.F. $\frac{0}{0}$

b. $\lim_{x\to 0} \dfrac{\sin(x)-x}{e^x-x-1} \overset{L'H}{=} \lim_{x\to 0} \dfrac{\cos(x)-1}{e^x-1} = \lim_{x\to 0} \dfrac{-\sin(x)}{e^x} = \dfrac{0}{1} = \boxed{0}$

I.F. $\frac{0}{0}$ I.F. $\frac{0}{0}$

c. $\lim_{x\to+\infty} e^{-x}x^2 = \lim_{x\to\infty} \dfrac{x^2}{e^x} \overset{L'H}{=} \lim_{x\to\infty} \dfrac{2x}{e^x} \overset{L'H}{=} \lim_{x\to\infty} \dfrac{2}{e^x} = \boxed{0}$ (0)

I.F. $0\cdot\infty$ I.F. $\frac{\infty}{\infty}$ I.F. $\frac{\infty}{\infty}$

3. [6 points] Evaluate the following antiderivatives. You do not need to simplify your final answers.

a. $\int (15x^4+6-2\csc^2(x))\,dx = \boxed{3x^5+6x+2\cot(x)+C}$

b. $\int \dfrac{x-\sqrt{x}}{x^2}dx = \int \left(\dfrac{1}{x}-x^{-3/2}\right)dx = \boxed{\ln|x|+2x^{-1/2}+C}$