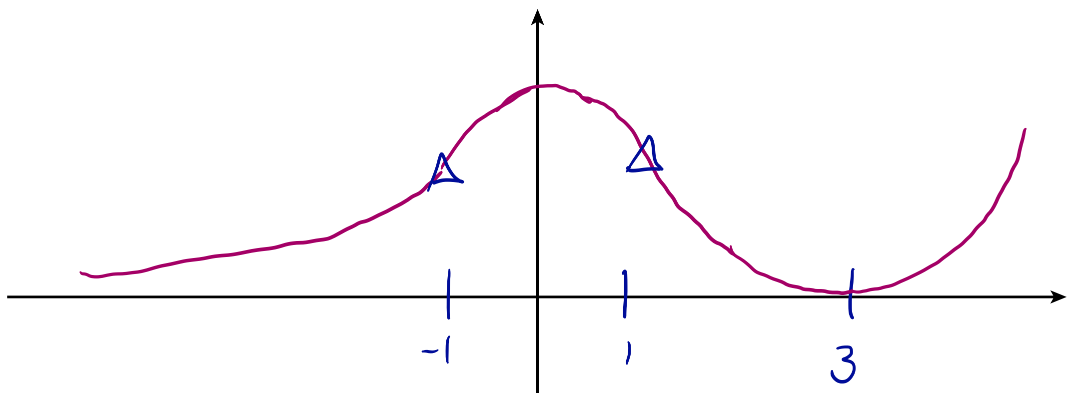
Name: Solutions ______ / **30**

There are 30 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [10 points] Sketch the graph of a continuous function with domain $\mathbb{R}$ that satisfies all of the following features.

1. $f(3) = 0$,
2. $f'(x) > 0$ for $x < 0$; $f'(x) < 0$ for x in $(0,3)$; $f'(x) > 0$ for $x > 0$,
3. $f'(0) = f'(3) = 0$,
4. $f''(x) < 0$ for $-1 < x < 1$; $f''(x) > 0$ for $x < -1$ or $x > 1$
5. $\lim_{x\to-\infty} = 0$; $\lim_{x\to\infty} f(x) = \infty$

Your sketch should label all interesting points on the x-axis. Additionally, place a **small triangle** on the graph at any points of inflection.



2. [6 points] Compute the following limits.

a. $\lim_{x\to 1} \frac{x^a - 1}{x^{2b} - 1}$ where a and b are constants, $b \neq 0$.

$$\lim_{x\to 1} \frac{x^a - 1}{x^{2b}-1} \overset{0/0}{=} \lim_{x\to 1} \frac{a x^{a-1}}{2b\, x^{2b-1}} = \frac{a}{2b}$$

b. $\lim_{x\to\infty} x^2 e^{-3x}$.

$$\lim_{x\to\infty} x^2 e^{-3x} = \lim_{x\to\infty} \frac{x^2}{e^{3x}} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{2x}{3e^{3x}} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{2}{9e^{3x}} = 0$$

3. [6 points] Consider the function $f(x) = \frac{1}{x} + \ln x$. We have computed for you

$$f'(x) = \frac{x-1}{x^2}; \qquad f''(x) = \frac{2-x}{x^{\cancel{2}3}}.$$

a. Find the intervals where $f(x)$ is increasing and decreasing. [Be careful about the domain of $f(x)$!]

$f'(x) = \frac{x-1}{x^2}$ ← determines sign; x^2 ← > 0

f': 0 — 1 +

decreases on $(0,1)$, inc on $(1,\infty)$

b. Find the intervals where $f(x)$ is concave up and concave down.

$f''(x) = \frac{2-x}{x^3}$ ← determines sign; x^3 ← > 0 on $(0,\infty)$

f'': 0 + 2 —

concave up: $(0,2)$, concave down $(2,\infty)$

4. [8 points] Consider the function $f(x) = x \ln x$.

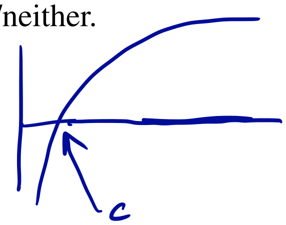
a. This function has a single critical point c. Find it.

$f'(x) = \ln(x) + \frac{x}{x} = \ln(x) + 1$

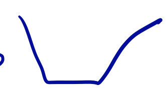
$f'(c) = 0 \Rightarrow c = e^{-1}$

b. Use the First Derivative Test to classify the critical point c from part **a**) as a local minimum/maximum/neither.

$\ln(x)+1$:



f': 0 — c + $\Rightarrow$



c is location of a local min

c. Use the Second Derivative Test to classify the critical point c from part **a**) as a local minimum or maximum if this is possible (or state that the Second Derivative Test is inconclusive).

$f''(x) = \frac{1}{x} > 0$ on $(0,\infty)$

$f''(c) > 0 \Rightarrow$ c is location of a local min.