

Name: Solutions

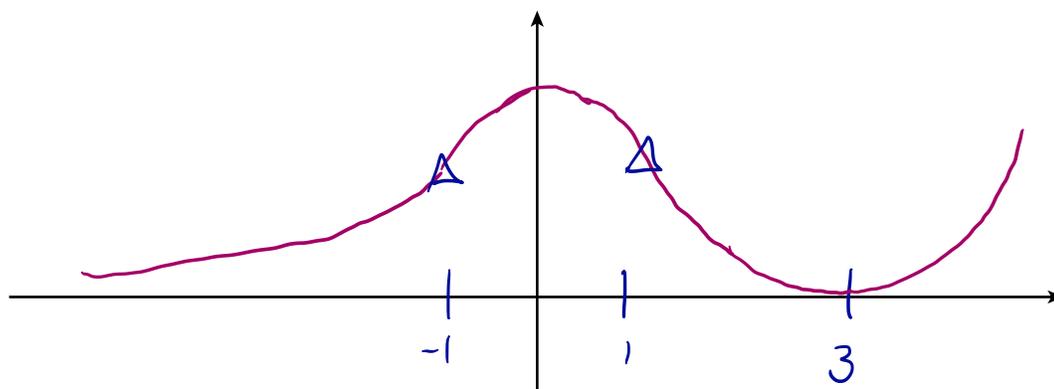
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There are 30 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [10 points] Sketch the graph of a continuous function with domain  $\mathbb{R}$  that satisfies all of the following features.

1.  $f(3) = 0$ ,
2.  $f'(x) > 0$  for  $x < 0$ ;  $f'(x) < 0$  for  $x$  in  $(0, 3)$ ;  $f'(x) > 0$  for  $x > 3$ ,
3.  $f'(0) = f'(3) = 0$ ,
4.  $f''(x) < 0$  for  $-1 < x < 1$ ;  $f''(x) > 0$  for  $x < -1$  or  $x > 1$
5.  $\lim_{x \rightarrow -\infty} f(x) = 0$ ;  $\lim_{x \rightarrow \infty} f(x) = \infty$

Your sketch should label all interesting points on the  $x$ -axis. Additionally, place a **small triangle** on the graph at any points of inflection.



2. [6 points] Compute the following limits.

a.  $\lim_{x \rightarrow 1} \frac{x^a - 1}{x^{2b} - 1}$  where  $a$  and  $b$  are constants,  $b \neq 0$ .

$$\lim_{x \rightarrow 1} \frac{x^a - 1}{x^{2b} - 1} = \frac{0}{0} = \lim_{x \rightarrow 1} \frac{ax^{a-1}}{2bx^{2b-1}} = \frac{a}{2b}$$

b.  $\lim_{x \rightarrow \infty} x^2 e^{-3x}$ .

$$\lim_{x \rightarrow \infty} x^2 e^{-3x} = \lim_{x \rightarrow \infty} \frac{x^2}{e^{3x}} = \frac{\infty}{\infty} = \lim_{x \rightarrow \infty} \frac{2x}{3e^{3x}} = \lim_{x \rightarrow \infty} \frac{2}{9e^{3x}} = 0$$

3. [6 points] Consider the function  $f(x) = \frac{1}{x} + \ln x$ . We have computed for you

$$f'(x) = \frac{x-1}{x^2}; \quad f''(x) = \frac{2-x}{x^2}$$

a. Find the intervals where  $f(x)$  is increasing and decreasing. [Be careful about the domain of  $f(x)$ !]

$f'(x) = \frac{x-1}{x^2}$  ← determines sign

$x^2 < > 0$

decreases on  $(0, 1)$ , inc on  $(1, \infty)$

A horizontal line with tick marks at 0 and 1. The region between 0 and 1 is labeled with a minus sign (-), and the region to the right of 1 is labeled with a plus sign (+). Arrows point from the text 'decreases on (0, 1), inc on (1, ∞)' to these regions.

b. Find the intervals where  $f(x)$  is concave up and concave down.

$f''(x) = \frac{2-x}{x^2}$  ← determines sign

$x^2 < > 0$  on  $(0, \infty)$

A horizontal line with tick marks at 0 and 2. The region between 0 and 2 is labeled with a plus sign (+), and the region to the right of 2 is labeled with a minus sign (-).

4. [8 points] Consider the function  $f(x) = x \ln x$ .

concave up:  $(0, 2)$ , concave down  $(2, \infty)$

a. This function has a single critical point  $c$ . Find it.

$$f'(x) = \ln(x) + \frac{x}{x} = \ln(x) + 1$$

$$f'(c) = 0 \Rightarrow c = e^{-1}$$

b. Use the First Derivative Test to classify the critical point  $c$  from part a) as a local minimum/maximum/neither.

$\ln(x) + 1$

A graph of the function  $f'(x) = \ln(x) + 1$  on a coordinate plane. The x-axis is labeled with 0 and  $c$ . The curve crosses the x-axis at  $x=c$ . To the left of  $c$ , the curve is below the x-axis (negative), and to the right, it is above (positive). An arrow points to the root at  $c$ .

$f'$  - | +

0 c

$\Rightarrow$

$c$  is location of a local min

c. Use the Second Derivative Test to classify the critical point  $c$  from part a) as a local minimum or maximum if this is possible (or state that the Second Derivative Test is inconclusive).

$$f''(x) = \frac{1}{x} > 0 \text{ on } (0, \infty)$$

$$f''(c) > 0 \Rightarrow c \text{ is location of a local min.}$$