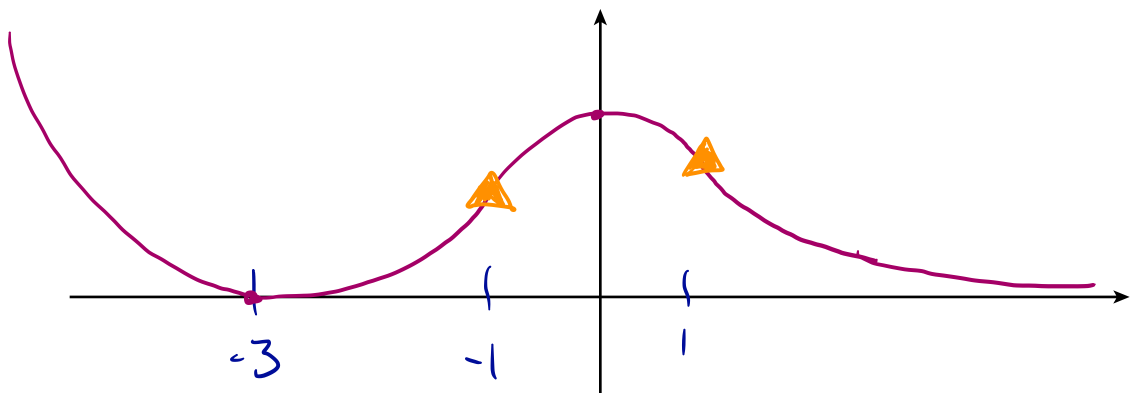
Name: Solutions ______ / 30

There are 30 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [10 points] Sketch the graph of a continuous function with domain $\mathbb{R}$ that satisfies all of the following features.

1. $f(-3) = 0$,
2. $f'(x) < 0$ for $x < -3$; $f'(x) > 0$ for x in $(-3, 0)$; $f'(x) < 0$ for $x > 0$,
3. $f'(-3) = f'(0) = 0$,
4. $f''(x) < 0$ for $-1 < x < 1$; $f''(x) > 0$ for $x < -1$ or $x > 1$
5. $\lim_{x\to-\infty} = \infty$; $\lim_{x\to\infty} f(x) = 0$

Your sketch should label all interesting points on the x-axis. Additionally, place a **small triangle** on the graph at any points of inflection.



2. [6 points] Compute the following limits.

a. $\lim_{x\to 1} \frac{x^{2a}-1}{x^b-1}$ where a and b are constants, $b \neq 0$.

$$\lim_{x\to 1} \frac{x^{2a}-1}{x^b-1} \overset{\frac{0}{0}}{=} \lim_{x\to 1} \frac{2a\,x^{2a-1}}{b\,x^{b-1}} = \frac{2a}{b}$$

b. $\lim_{x\to\infty} x^2 e^{-4x}$.

$$\lim_{x\to\infty} x^2 e^{-4x} = \lim_{x\to\infty} \frac{x^2}{e^{4x}} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{2x}{4e^{4x}} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to\infty} \frac{2}{16e^{x}} = 0$$

3. [6 points] Consider the function $f(x) = \frac{2}{x} + \ln x$. We have computed for you

$$f'(x) = \frac{x-2}{x^2}; \qquad f''(x) = \frac{4-x}{x^3}.$$

a. Find the intervals where $f(x)$ is increasing and decreasing. [Be careful about the domain of $f(x)$!]

$f'(x) = \frac{x-2}{x^2}$ $x-2 \to$ controls sign, $x^2 \to >0$

f': 0 $-$ 2 $+$

$(0,2)$: increasing
$(2,\infty)$: decreasing

b. Find the intervals where $f(x)$ is concave up and concave down.

$f''(x)$: $\frac{4-x}{x^3}$ $4-x \leftarrow$ controls sign, $x^3 \leftarrow >0$ on domain $(0,\infty)$

f'': 0 $+$ 4 $-$

$(0,4)$: concave up
$(4,\infty)$: concave down

4. [8 points] Consider the function $f(x) = x\ln x$.

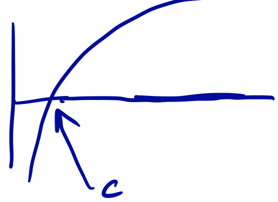
a. This function has a single critical point c. Find it.

$f'(x) = \ln(x) + \frac{x}{x} = \ln(x) + 1$

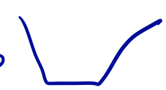
$f'(c) = 0 \Rightarrow c = e^{-1}$

b. Use the First Derivative Test to classify the critical point c from part **a**) as a local minimum/maximum/neither.

$\ln(x)+1$:



f': 0 $-$ c $+$ $\Rightarrow$



c is location of a local min

c. Use the Second Derivative Test to classify the critical point c from part **a**) as a local minimum or maximum if this is possible (or state that the Second Derivative Test is inconclusive).

$f''(x) = \frac{1}{x} > 0$ on $(0,\infty)$

$f''(c) > 0 \Rightarrow$ c is location of a local min.