Math 251: Quiz 5 **February 26, 2019**

Name: Solutions ______ / 25

Circle one: **Rhodes (F01) | Bueler (F02)**

25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. **[15 points]** Differentiate the following. Use proper notation to indicate your answer.

a. $f(x) = \sqrt{5+\sin x} = (5+\sin x)^{1/2}$

$$f'(x) = \frac{1}{2}(5+\sin x)^{-1/2}(\cos x)$$
$$= \frac{\cos x}{2\sqrt{5+\sin x}}$$

b. $f(x) = e^{x\tan x}$

$$\frac{df}{dx} = e^{x\tan x}\left(\tan x + x\sec^2 x\right)$$

c. $g(x) = \sec^2(3x)$

$$g'(x) = 2\sec(3x)\sec(3x)\tan(3x)\,3$$
$$= 6\sec^2(3x)\tan(3x)$$

d. $y = x2^x = x e^{x\ln 2}$

$$\frac{dy}{dx} = e^{x\ln 2} + x e^{x\ln 2}\ln 2$$
$$= 2^x + x2^x\ln 2$$

e. $f(\theta) = \theta e^\theta \cos\theta$

$$f'(\theta) = e^\theta\cos\theta + \theta\frac{d}{d\theta}(e^\theta\cos\theta)$$
$$= e^\theta\cos\theta + \theta(e^\theta\cos\theta - e^\theta\sin\theta)$$

2. [4 points] An object is at position $s(t) = \sqrt{t^2 - 6t + 11}$ meters at time $t \geq 0$ seconds. When, if ever, is its instantaneous velocity 0?

$s(t) = (t^2 - 6t + 11)^{1/2}$

$s'(t) = \frac{1}{2}(t^2 - 6t + 11)^{-1/2}(2t - 6) = 0$

$2t - 6 = 0$

$t = 3$

3. [6 points] Find an equation of the tangent line to the curve $y = \dfrac{2}{(1+\sin x)^3}$ at the point where $x = \pi$.

$y = 2(1+\sin x)^{-3}$

$y\big|_{x=\pi} = 2(1+0)^{-3} = 2$

$\frac{dy}{dx} = -6(1+\sin x)^{-4}(\cos x)$

$\frac{dy}{dx}\Big|_{x=4} = -6(1+0)^{-4}(-1) = 6$

$y - 2 = 6(x - \pi)$

$y = 2 + 6(x - \pi)$