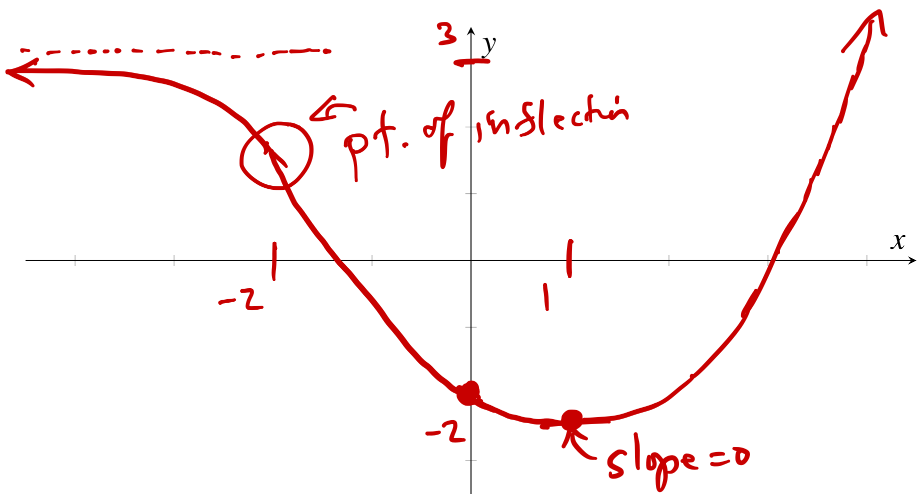
Name: ______________________________ ________ / 25

Circle one: **Rhodes (F01)** | **Bueler (F02)**

25 points possible. No aids (book, calculator, etc.) are permitted. You need not simplify, but show all work and use proper notation for full credit.

1. [8 points] Sketch an appropriately labeled graph of a function that satisfies all of the given conditions.

1. $f(0) = -2$
2. $f'(1) = 0$
3. $f'(x) < 0$ for $x < 1$; $f'(x) > 0$ for $x > 1$
4. $f''(x) < 0$ for $x < -2$; $f''(x) > 0$ for $x > -2$
5. $\lim_{x\to-\infty} f(x) = 3$; $\lim_{x\to\infty} f(x) = \infty$



2. [4 points] Compute the following limits.

a. $\lim_{x\to\infty} \dfrac{\sqrt{x}}{e^{3x}}$ $\underset{L'H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty} \dfrac{\frac{1}{2}x^{-1/2}}{3e^{3x}} = \lim_{x\to\infty} \dfrac{1}{6\sqrt{x}\,e^{3x}} = 0$

b. $\lim_{x\to 0} \dfrac{x^2}{2-e^x} = \dfrac{0}{2-e^0} = \dfrac{0}{2-1} = 0$

3. [13 points] Consider the function $f(x) = \ln(x^2+4)$. We have computed for you

$$f'(x) = \frac{2x}{x^2+4}, \qquad f''(x) = \frac{-2x^2+8}{(x^2+4)^2}.$$

a. Find the domain of $f(x)$.

$x^2+4>0$ so: $(-\infty,\infty)$

b. Find intercepts.

$x^2+4 \geq 4 > 1$ so no x-intercepts

$y = \ln(4)$ is y-intercept

c. Find the critical point(s).

$\frac{2x}{x^2+4} = 0$ $\quad x=0$ is critical number

d. Determine the intervals where $f(x)$ is increasing and decreasing.

increasing on $[0,\infty)$

decreasing on $(-\infty,0]$

e. Find the intervals where $f(x)$ is concave up and concave down.

$-2x^2+8=0$

$x^2=4$

$x=\pm 2$

Concave up on $(-2,2)$

Concave down on $(-\infty,-2) \cup (2,\infty)$

f. Using the above information, sketch the graph of $f(x)$, making sure to label x-coordinates of all important points. [Hint: $\ln 4 \approx 1.5$]