Math 251: Quiz 3 **4 February 2020**

Name: Solutions **______ / 25**

Circle one: **Faudree (F01)** | **Bueler (F02)** | **VanSpronsen (UX1)**

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [9 points] Evaluate each limit below. Your answer for each should be either a real number, $+\infty$, $-\infty$, or DNE. Show your work to receive full credit.

a. $\lim_{x\to -3} \frac{x^2+4x+3}{x^2+x-6}$. $= \cancel{\frac{9-12+3}{9-3-6} = \frac{0}{0}}$ ← factor + cancel!

$$= \lim_{x\to -3} \frac{(x+3)(x+1)}{(x+3)(x-2)} = \lim_{x\to -3} \frac{x+1}{x-2} = \frac{-3+1}{-3-2} = \frac{-2}{-5} = \frac{2}{5}$$

b. $\lim_{x\to 9} \frac{3-\sqrt{x}}{9x-x^2}$ $= \cancel{\frac{0}{0}}$ Do algebra!

$$\lim_{x\to 9} \frac{3-\sqrt{x}}{9x-x^2} \cdot \frac{3+\sqrt{x}}{3+\sqrt{x}} = \lim_{x\to 9} \frac{9-x}{x(9-x)(3+\sqrt{x})} = \lim_{x\to 9} \frac{1}{x(3+\sqrt{x})} = \frac{1}{9(3+3)} = \frac{1}{54}$$

c. $\lim_{h\to 0^-} \frac{2h^2+10h}{|h|}$ $= \lim_{h\to 0^-} \frac{2h(h+5)}{|h|} = \lim_{h\to 0^-} \frac{2h(h+5)}{-h} = \lim_{h\to 0^-} -2(h+5)$

because if $h<0$, then $|h| = -h$.

$$= -2(0+5) = -10$$

2. [4 points] Use the Intermediate Value Theorem to **show** that the equation $e^x = 4-5x$ has a root in the interval $(0,1)$.

Let $f(x) = e^x - 4 + 5x$. Observe that $f(x)$ is continuous.

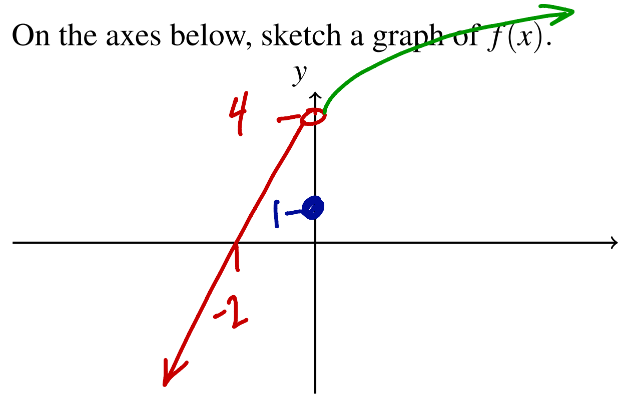
Now $f(0) = e^0 - 4 + 5\cdot 0 = -3 < 0$ and $f(1) = e^1 - 4 + 5 = e+1 > 0$.

Since f is negative at $x=0$ and positive at $x=1$, it must be zero somewhere in between.

3. [8 points] Consider the function $f(x) = \begin{cases} -2x+4 & x<0 \\ 1 & x=0 \\ \sqrt{x+16} & x>0. \end{cases}$ ✓ ✓ ✓

$x=0$: $-2\cdot 0+4=4$

a. On the axes below, sketch a graph of $f(x)$.



b. Evaluate the limit below or explain why the limit fails to exist.

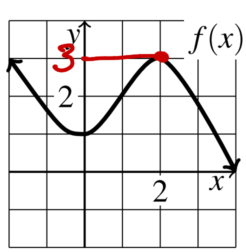
$\lim_{x\to 0} f(x) = 4$ because $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} 2x+4 = 4$ and

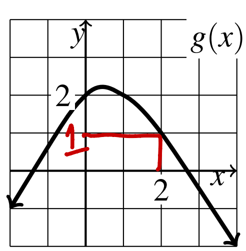
$\lim_{x\to 0^-} f(x) = \lim_{x\to 0} \sqrt{x+16} = 4.$

c. Is f continuous at $x=0$? Explain using the definition of continuity.

No. $\lim_{x\to 0} f(x) = 4 \neq 1 = f(0)$

4. [4 points] The graphs of $f(x)$ and $g(x)$ are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.





a. $\lim_{x\to 2}\left(\frac{5f(x)}{2+g(x)}\right) = \frac{5\cdot 3}{2+1} = \frac{15}{3} = 5$

b. $\lim_{x\to 2}\left(x^2 f(x)\right) = 2^2 \cdot 3 = 12$