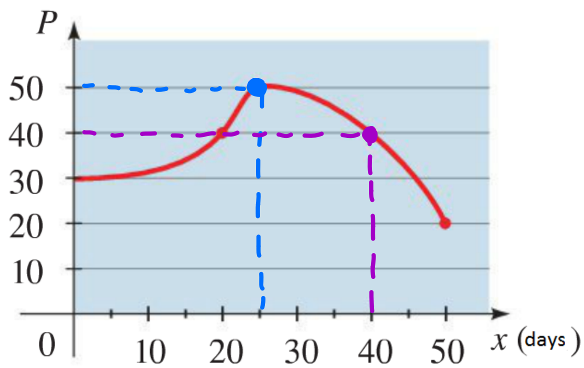
Name: Solutions ______ / 25

No aids (calculator, notes, text, etc.) are permitted. Show all work for full credit.

1. [5 points] The graph below shows the population P of mice in a particular garden over the course of 50 days. Give answers to the following in correct units.



a. Find the number of mice on days 25 and 40.

$P(25) = 50$ (mice) $P(40) = 40$ (mice)

b. Find the average rate of change of the population from $x = 25$ to $x = 40$.

$$v_{average} = \frac{\Delta P}{\Delta x} = \frac{P(40)-P(25)}{40-25} = \frac{40-50}{15} = -\frac{10}{15} = -\frac{2}{3} \text{ (mice/day)}$$

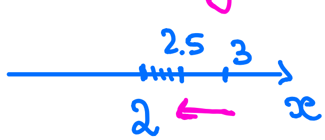
c. Find the average rate of change of the population during the entire period. $[0,50]$

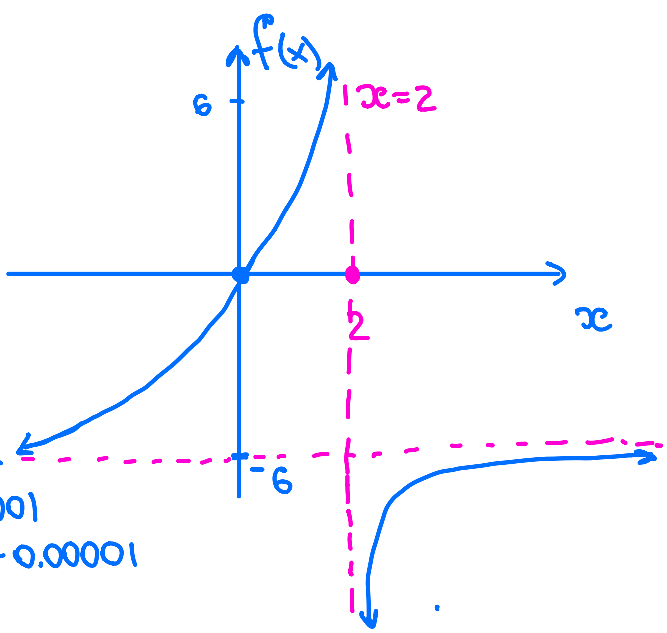
$$v_{average} = \frac{\Delta P}{\Delta x} = \frac{P(50)-P(0)}{50-0} = \frac{20-30}{50} = -\frac{10}{50} = -\frac{1}{5} \text{ (mice/day)}$$

2. [6 points] Compute the following limit. Justify your answer with a sentence or two.

$$\lim_{x\to 2^+} \frac{6x}{2-x} = \boxed{-\infty}$$

$$\lim_{x\to 2^+} \frac{6x}{\underbrace{2-x}_{\to 0^-}} = \frac{12}{0^-} = -\infty$$





$2-2.5 = -0.5$

$2-2.1 = -0.1$

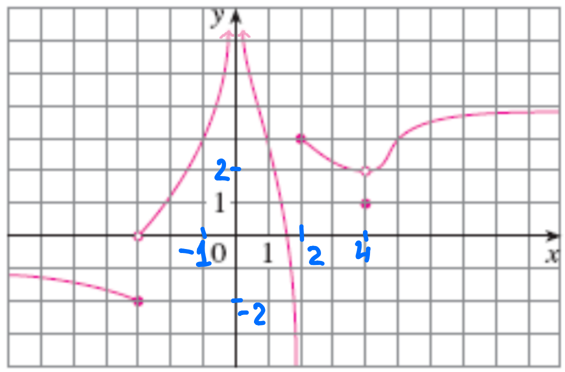
$2-2.001 = -0.001$

$2-2.00001 = -0.00001$

.....

$$f(x) = \frac{6x}{2-x} = \frac{6x-12+12}{2-x} = \frac{-6(2-x)+12}{2-x} = -6 + \frac{12}{2-x}$$

3. [9 points] Use the graph of the function of $f(x)$ to answer the following questions.



a. $\lim_{x\to 4} f(x) =$ 2

b. $\lim_{x\to 2^-} f(x) =$ $-\infty$

c. $\lim_{x\to -1} f(x) =$ 3

d. $f(-1) =$ 3

e. $f(4) =$ 1

f. $f(-3) =$ -2

g. $\lim_{x\to -3^-} f(x) =$ -2

h. $\lim_{x\to -3^+} f(x) =$ 0

i. $\lim_{x\to -3} f(x) =$ DNE

4. [5 points] Suppose the distance traveled by a car from time $t = 0$ minutes is given by $d(t) = t + t^2$ where distance is measured in miles.

a. Compute the average speed from time $t = 1$ to time $t = 3$ minutes.

$$v_{average} = \frac{\Delta d}{\Delta t} = \frac{d(3) - d(1)}{3-1} = \frac{(3+9)-(1+1)}{2} = \frac{10}{2} = 5 \text{ (miles/min)}$$

In general:

$$v_{average} = \frac{\Delta d}{\Delta t} = \frac{d(t_2) - d(t_1)}{t_2 - t_1} \quad \text{on } [t_1, t_2]$$

b. Compute the average speed from time $t = 1$ to time $t = 2$ minutes.

$$v_{average} = \frac{\Delta d}{\Delta t} = \frac{d(2) - d(1)}{2-1} = \frac{(2+4)-(1+1)}{1} = \frac{4}{1} = 4 \text{ (miles/min)}$$

c. What goes wrong in the previous computations if you try to compute the exact speed at time $t = 1$ minutes by computing an average speed from time $t = 1$ to time $t = 1$?

$$v(1) = \frac{\Delta d}{\Delta t} = \frac{d(1) - d(1)}{1-1} = \frac{0}{0} \rightarrow \text{undefined}$$

We can ask what happens as $t \to 1$ but we can't plug in $t = 1$.