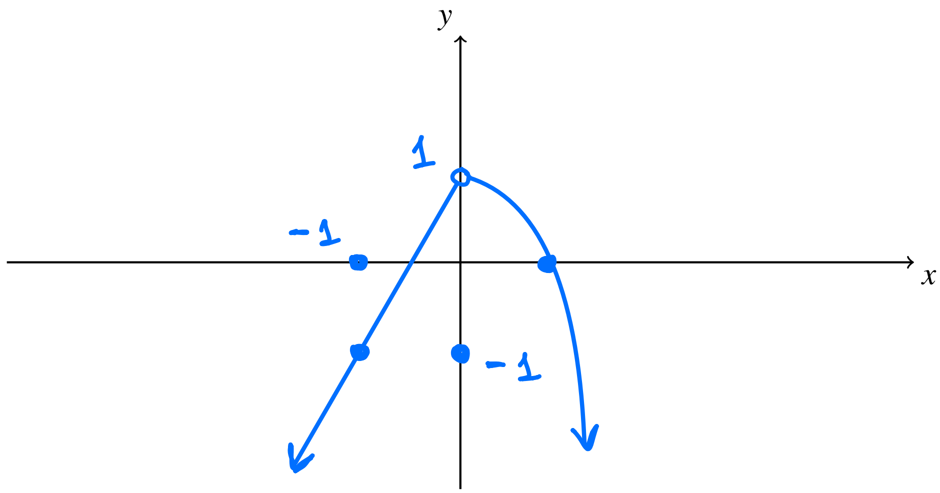
Name: Solutions ______ / 20

20 points possible. No aids (book, calculator, etc.) are permitted. Show all work for full credit.

1. [6 points] Consider the function $f(x) = \begin{cases} 1+2x & \text{if } x<0 \\ -1 & \text{if } x=0 \\ 1-x^2 & \text{if } x>0 \end{cases}$.

a. On the axes below, sketch a graph of $f(x)$.



b. Evaluate (**with justification**) the limit, or explain why it does not exist:

$\lim_{x\to 0} f(x) = 1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = 1$

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} (1-x^2) = 1$

direct substitution

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} (1+2x) = 1$

c. Is f continuous at $x=0$? Explain using the **definition** of continuity.

$y=f(x)$ is not continuous at $x=0$ since

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = 1 \neq f(0) = -1$.

2. [4 points] Use the Intermediate Value Theorem to show that there is a root of the equation $x - 2\cos(x) + 1 = 0$ in the interval $(0, \pi)$.

Let $f(x) = x - 2\cos(x) + 1$ (is continuous on $[0, \pi]$)

- $f(0) = 0 - 2\cos(0) + 1 = -2 + 1 = \underline{-1 < 0}$
- $f(\pi) = \pi - 2\cos(\pi) + 1 = \underline{\pi - 1 > 0}$

Therefore, there exists a number c in $(0, \pi)$ such that $\boxed{f(c) = 0}$

3. [6 points] Evaluate the limit. Show work and **use proper limit notation** for full credit.

$$\lim_{h\to 0}\frac{\frac{1}{5+h}-\frac{1}{5}}{h} = \text{"}\frac{0}{0}\text{"} \text{ type (Needs algebra)}$$

$$\lim_{h\to 0}\frac{\frac{1}{5+h}-\frac{1}{5}}{h} = \lim_{h\to 0}\frac{\cancel{5}-\cancel{5}-h}{5(h+5)\cdot h} = \lim_{h\to 0}\frac{-\cancel{h}}{5(h+5)\cancel{h}} = -\frac{1}{25}$$

4. [4 points]

a. Why is the following not, strictly speaking, a fully true statement?:

$$\frac{(x-1)(x+2)}{x-1} = x+2$$

Let $f(x) = x+2$ and $g(x) = \frac{(x-1)(x+2)}{x-1}$

$\mathrm{Dom}(f) = \mathbb{R}$

$\neq$

$\mathrm{Dom}(g) = \mathbb{R}\setminus\{1\}$

$\Rightarrow$ The statement is not fully true.

b. Carefully sketch the graph of $f(x) = \frac{(x-1)(x+2)}{x-1}$ on the interval $[0,2]$.