Name: ______________________________ ________ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[15 points]** Find the derivative for each function below. You do not need to simplify. You do need to use parentheses correctly.

a. $h(x) = 2^x + \log_2(x)$

$h'(x) = (\ln 2)\,2^x + \frac{1}{(\ln 2)x}$

b. $f(x) = \sin^{-1}(\sqrt{x}) = \sin^{-1}(x^{1/2})$

$f'(x) = \frac{1}{\sqrt{1-(x^{1/2})^2}} \cdot \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}\,\sqrt{1-x}}$

c. $y = \left(x^{-1} + \tan^{-1}(x)\right)^3$

$\frac{dy}{dx} = 3\left(x^{-1} + \tan^{-1}(x)\right)^2\left(-x^{-2} + \frac{1}{1+x^2}\right)$

d. $g(x) = \frac{x^2+1}{e^{2x}} = (x^2+1)e^{-2x}$ ⟶ OR Product rule

$g'(x) = 2xe^{-2x} + (x^2+1)\cdot e^{-2x}(-2)$

$= e^{-2x}\left(2x - 2(x^2+1)\right)$

quotient rule:

$g'(x) = \frac{(e^{2x})(2x) - (x^2+1)e^{2x}(2)}{(e^{2x})^2}$

$= \frac{e^{2x}\left(2x - 2(x^2+1)\right)}{(e^{2x})^2} = \frac{2x - 2(x^2+1)}{e^{2x}}$

e. $y = 5x^{4/3} + \ln(5x^{4/3}) = 5x^{4/3} + \ln 5 + \ln(x^{4/3}) = 5x^{\frac{4}{3}} + \ln 5 + \frac{4}{3}\ln x$

OR Simplify using algebra:

derivative as is:

$\frac{dy}{dx} = 5\cdot\frac{4}{3}\cdot x^{1/3} + \frac{1}{5x^{4/3}}\cdot 5\left(\frac{4}{3}\right)x^{1/3}$

$= \frac{20}{3}x^{1/3} + \frac{4}{3x}$

$\frac{dy}{dx} = 5\cdot\frac{4}{3}x^{\frac{1}{3}} + 0 + \frac{4}{3}\cdot\frac{1}{x}$

$= \frac{20}{3}x^{\frac{1}{3}} + \frac{4}{3x}$

2. [5 points] Use implicit differentiation to find $\frac{dy}{dx}$ for $x^2 + y^2 = \cos(xy) + 2$.

$$2x + 2y\frac{dy}{dx} = -\cos(xy)\left[1\cdot y + x\cdot\frac{dy}{dx}\right] + 0$$

distribute

$$2x + 2y\frac{dy}{dx} = -y\cos(xy) - x\cos(xy)\frac{dy}{dx}$$

$$\left[2y + x\cos(xy)\right]\frac{dy}{dx} = -y\cos(xy) - 2x$$

$$\frac{dy}{dx} = \frac{-y\cos(xy) - 2x}{2y + x\cos(xy)}$$

3. [5 points] Use logarithmic differentiation to find $\frac{dy}{dx}$ for $y = (\sin(2x))^x$.

$$y = (\sin(2x))^x$$

$$\ln y = \ln\left((\sin(2x))^x\right) = x\ln(\sin(2x))$$

$$\frac{1}{y}\frac{dy}{dx} = 1\cdot\ln(\sin(2x)) + x\cdot\frac{1}{\sin(2x)}\cdot\cos(2x)\cdot 2$$

$$\frac{dy}{dx} = y\left(\ln(\sin(2x)) + \frac{2x\cos(2x)}{\sin(2x)}\right)$$

$$= \left((\sin(2x))^x\right)\left(\ln(\sin(2x)) + \frac{2x\cos(2x)}{\sin(2x)}\right)$$