Name: Solutions ______ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. (15 points) Find the derivative of each function. You do not need to simplify your answer.

   (a) $g(\theta) = 5\arcsin(2\theta)$

   $$g'(\theta) = 5 \cdot \frac{1}{\sqrt{1-(2\theta)^2}} \cdot (2) = \frac{10}{\sqrt{1-4\theta^2}}$$

   (b) $f(x) = e^x \tan^{-1}(x)$

   $$f'(x) = e^x \cdot \tan^{-1}(x) + e^x \cdot \left(\frac{1}{1+x^2}\right) = e^x\left(\tan^{-1}(x) + \frac{1}{1+x^2}\right)$$

   (c) $x(t) = \ln(t^3+1)$

   $$x'(t) = \frac{1}{t^3+1} \cdot (3t^2) = \frac{3t^2}{t^3+1}$$

   (d) $f(x) = x^{2/3} + e^{-3x}$

   $$f'(x) = \frac{2}{3} x^{-1/3} - 3e^{-3x}$$

   (e) $h(x) = e^2 + (\cos(x))^{-1}$

   $$h(x) = 0 + (-1)(\cos(x))^{-2}(-\sin(x))$$
   $$= \sin(x)(\cos(x))^{-2}$$

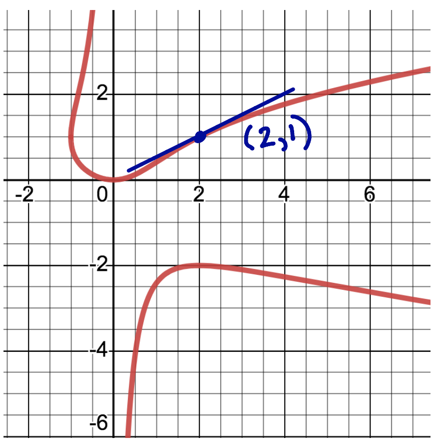
2. (4 points) **Use logarithmic differentiation** to find $\frac{dy}{dx}$ for the function $y(x) = \frac{x\ \sin^2(\ x)}{x^2+5}$.

$$\ln(y) = \ln(x) + 2\ln(\sin(x)) - \ln(x^2+5)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{x} + \frac{2}{\sin(x)}\cdot\cos(x) - \frac{1}{x^2+5}\cdot 2x$$

$$\frac{dy}{dx} = (y)\left(\frac{1}{x} + \frac{2\cos(x)}{\sin(x)} - \frac{2x}{x^2+5}\right) = \frac{x\sin^2(x)}{x^2+5}\left(\frac{1}{x} + \frac{2\cos(x)}{\sin(x)} - \frac{2x}{x^2+5}\right)$$

3. (6 points) The graph of the equation $xy^2 = x^2 - 2y$ is drawn below.



(a) Use implicit differentiation to find $\frac{dy}{dx}$.

$$1\cdot y^2 + x\cdot 2y\frac{dy}{dx} = 2x - 2\frac{dy}{dx}$$

$$2xy\frac{dy}{dx} + 2\frac{dy}{dx} = 2x - y^2$$

$$\frac{dy}{dx} = \frac{2x-y^2}{2xy+2}$$

(b) This curve has two different points when $x = 2$. Find the equation of line tangent to the curve at $x = 2$ **in the first quadrant**. Draw the tangent line on the graph above.

Check that (2,1) is on graph: $2\cdot 1^2 = 2 = 2^2 - 2\cdot 1$ ✓

$$\left.\frac{dy}{dx}\right|_{(2,1)} = \frac{2(2)-1^2}{2(2)(1)+2} = \frac{3}{6} = \frac{1}{2} = m$$

line: $y - 1 = \frac{1}{2}(x-2)$ or $y = 1 + \frac{1}{2}(x-2)$ or $y = \frac{1}{2}x$