Name: Key ______ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. [12 points] Evaluate the following limits. If a value does not exist, write DNE. You must show work to receive full credit.

a. $\lim_{x\to 3}\frac{2x^2-6x}{x^2+x-12} = \lim_{x\to 3}\frac{2x(x-3)}{(x-3)(x+4)} = \lim_{x\to 3}\frac{2x}{x+4} = \frac{2(3)}{3+4} = \boxed{\frac{6}{7}}$

b. $\lim_{\substack{\cancel{h}\to 0\\ x}}\frac{x^3-4x}{(1+\sin x)\cos x} = \frac{0^3-4(0)}{(1+\sin(0))\cos(0)} = \frac{0}{1\cdot 1} = \boxed{0}$

c. $\lim_{x\to 2}\frac{\sqrt{7+x}-3}{x-2}\cdot\frac{\sqrt{7+x}+3}{\sqrt{7+x}+3} = \lim_{x\to 2}\frac{7+x-9}{(x-2)(\sqrt{7+x}+3)} = \lim_{x\to 2}\frac{x-2}{(x-2)(\sqrt{7+x}+3)}$

$= \lim_{x\to 2}\frac{1}{\sqrt{7+x}+3} = \frac{1}{\sqrt{9}+3} = \boxed{\frac{1}{6}}$

2. [4 points] Given $\lim_{x\to 10} f(x) = 2$ and $\lim_{x\to 10} g(x) = -5$, evaluate $\lim_{x\to 10} 2\left(\frac{x+1}{f(x)+g(x)}\right)$ using limit laws.

$\lim_{x\to 10} 2\left(\frac{x+1}{f(x)+g(x)}\right) = 2\left(\frac{\lim_{x\to 10} x + \lim_{x\to 10} 1}{\lim_{x\to 10} f(x) + \lim_{x\to 10} g(x)}\right) = 2\left(\frac{10+1}{2+(-5)}\right)$

$= \frac{22}{-3} = \boxed{-\frac{22}{3}}$

3. **[5 points]** Let $f(x) = \begin{cases} (x-1)^2 & x < 0 \\ e^x & x \geq 0 \end{cases}$.

a. Find $\lim\limits_{x \to 0^-} f(x)$.

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x \to 0^-} (x-1)^2 = (-1)^2 = \boxed{1}$

b. Find $\lim\limits_{x \to 0^+} f(x)$.

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x \to 0^+} e^x = e^0 = \boxed{1}$

c. What is $f(0)$?

$f(0) = e^0 = \boxed{1}$

d. Use your answers to parts (a), (b) and (c) to justify whether $f(x)$ is or is not continuous at $x = 0$. (Your answer should be a complete sentence.)

$f(x)$ is continuous at $x = 0$ since $\lim\limits_{x \to 0} f(x) = 1 = f(0)$.

4. **[4 points]** Use the Intermediate Value Theorem to show that $f(x) = \sin(2x) - \cos(3x) = 0$ for some x-value on the interval $(0, \pi)$.

Note that $f(x)$ is a continuous function.

$f(0) = \sin(0) - \cos(0) = 0 - 1 = -1 < 0$

$f(\pi) = \sin(2\pi) - \cos(3\pi) = 0 - (-1) = 1 > 0$

Therefore, by the IVT, $f(x) = 0$ for some x-value in $(0, \pi)$.