Name: Key ______ / 25

There are 25 points possible on this quiz. No aids (book, calculator, etc.) are permitted. **Show all work for full credit.**

1. **[10 points]** (Related Rate Problem) A spherical snowball is melting so that its volume is decreasing at a constant rate of $4\pi\, cm^3/min$. Use this information to answer the following questions.

   a. How fast is the **radius** of the snowball decreasing when the radius is $8\,cm$? Include units in your answer. (Use the fact that the volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.)

$$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{4}{3}\pi r^3\right]$$

$$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

$$4\pi = 4\pi(8)^2 \cdot \frac{dr}{dt}$$

$$\frac{1}{64} = \frac{dr}{dt}$$

The radius is decreasing at a rate of $\frac{1}{64}$ cm/min.

   b. How fast is the **surface area** of the snowball decreasing when the radius is $8\,cm$? Include units in your answer. (Use your answer in part (a) and that the surface area of a sphere is given by $S = 4\pi r^2$.)

$$\frac{d}{dt}[S] = \frac{d}{dt}[4\pi r^2]$$

$$\frac{dS}{dt} = 8\pi r \cdot \frac{dr}{dt}$$

$$\frac{dS}{dt} = 8\pi(8) \cdot \frac{1}{64} = \pi$$

The surface area is decreasing at a rate of π cm^2/min

2. [7 points] (Linear Approximation and Differentials) Let $f(x) = x^3 - \ln(x)$.

a. Find the linear approximation $L(x) = f(a) + f'(a)(x-a)$ to $y = f(x)$ at $a = 1$.

$f'(x) = 3x^2 - \frac{1}{x}$, $a = 1$

$f(1) = 1 - \ln(1) = 1$

$f'(1) = 3 - 1 = 2$

$L(x) = 1 + 2(x-1)$ or $L(x) = 2x - 1$

b. Use your linear approximation to estimate $f\left(\frac{3}{2}\right)$.

$L\left(\frac{3}{2}\right) = 2\left(\frac{3}{2}\right) - 1 = \boxed{2}$

3. [8 points] Let $h(x) = 4x^3 - 3x^4 + 6$.

a. Find all critical points for $h(x)$.

$h'(x) = 12x^2 - 12x^3 = 12x^2(1-x) \leftarrow h'$ is never undefined

Set $h'(x) = 0$: $12x^2(1-x) = 0 \Rightarrow x = 0,\ x = 1$ are critical points

b. Determine the absolute maximum and absolute minimum of $h(x)$ on the interval $[-1, 2]$ or state that none exist. You must show your work to receive full credit. See the answer-blank below.

x	$h(x)$
-1	-1
0	6
1	7
2	-10

maximum value of $h(x)$: 7

minimum value of $h(x)$: -10