

Name: Solutions

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Please circle your instructor's name:

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There are 25 points possible on this quiz. Any outside materials are not allowed. **For full credit, show all work clearly.**

1. [12 points] Evaluate the following definite integrals. Use proper notation and **simplify** your final answers.

$$\begin{aligned} \text{a. } \int_1^2 3x^2 - 2x^3 dx &= 3 \cdot \frac{1}{3} x^3 - 2 \cdot \frac{1}{4} x^4 \Big|_1^2 = x^3 - \frac{1}{2} x^4 \Big|_1^2 \\ &= \left(8 - \frac{1}{2} \cdot 16 \right) - \left(1 - \frac{1}{2} \cdot 1 \right) \\ &= -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{b. } \int_0^\pi 2 \cos(t) + \sin(t) dt &= 2 \sin(t) - \cos(t) \Big|_0^\pi \\ &= [2 \cdot 0 - (-1)] - [2 \cdot 0 - 1] \\ &= 1 + 1 = 2 \end{aligned}$$

$$\begin{aligned} \text{c. } \int_1^3 \frac{x^3 - 2x^2 + x}{x^2} dx &= \int_1^3 x - 2 + \frac{1}{x} dx = \frac{x^2}{2} - 2x + \ln|x| \Big|_1^3 \\ &= \left(\frac{9}{2} - 2 \cdot 3 + \ln(3) \right) - \left(\frac{1}{2} - 2 \cdot 1 + \ln(1) \right) \\ &= \frac{8}{2} - 4 + \ln(3) - 0 = \ln(3) \end{aligned}$$

2. [4 points] Suppose $F(x) = \int_1^{x^2} \ln(t) dt$. Find $F'(x)$.

$$F'(x) = \ln(x^2) \cdot 2x$$

3. [9 points] We define $A(x)$ to be the elevation in feet above sea level at mile x on the Appalachian Trail going from south to north. The distance from the beginning of the Appalachian Trail to the end is 2198 miles, and thus the domain of $A(x)$ is $[0, 2198]$.

- a. It is a fact that $A(0) = 3782$ and $A'(0) = -346$. Explain what these values mean in the context of the problem. Include units in your explanation.

$A(0)$: The starting elevation above sea level is 3,782 feet.

$A'(0)$: The slope of the trail at the start is going downhill at a rate of 346 ft/mile.

- b. Write a sentence interpreting the meaning of the value of $\int_0^{2198} A'(x) dx$ in the context of this problem. What are the units of this value?

This is the net change in the height of the trail from its beginning to its end.

- c. It is also a fact that $A(2198) = 5254$. With this information can you determine $\int_0^{2198} A'(x) dx$? If so, evaluate this integral. If not, explain why it is not possible.

By the Fundamental Theorem of the Calculus,

$$\int_0^{2198} A'(x) dx = A(2198) - A(0) = 5254 - 3782 = 1472 \text{ feet}$$

- d. (1 point extra credit!) What would the value of $\int_0^{2198} |A'(x)| dx$ represent?

This would determine the total change in elevation, without regard to direction of change, in other words, it would be the sum of the feet climbed plus the feet descended on the hike.