Name: Solutions ____________ / 25

Please circle your instructor's name: James Gossell Gordon Williams

There are 25 points possible on this quiz. Any outside materials are not allowed. **For full credit, show all work clearly.**

1. [10 points] The **derivative** of $f(x)$ is defined as follows:

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}.$$

a. Use the definition of the derivative (above) to find the derivative of the function $f(x) = \frac{2}{\sqrt{x}}$. Use proper limit notation. **No credit will be awarded for finding the derivative via other methods.**

$$f'(x) = \lim_{h\to 0} \frac{\frac{2}{\sqrt{x+h}} - \frac{2}{\sqrt{x}}}{h} = \lim_{h\to 0} \frac{\frac{2\sqrt{x} - 2\sqrt{x+h}}{\sqrt{x}\sqrt{x+h}}}{h}$$

$$= \lim_{h\to 0} \frac{2}{h} \cdot \frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x}\sqrt{x+h}} \cdot \frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}$$

$$= \lim_{h\to 0} \frac{2}{h} \frac{x-(x+h)}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})} = \lim_{h\to 0} \frac{2}{h} \cdot \frac{-h}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})}$$

$$= \lim_{h\to 0} \frac{-2}{\sqrt{x}\sqrt{x+h}(\sqrt{x}+\sqrt{x+h})} = \frac{-2}{\sqrt{x}\sqrt{x}(\sqrt{x}+\sqrt{x})} = \frac{-1}{x\sqrt{x}}$$

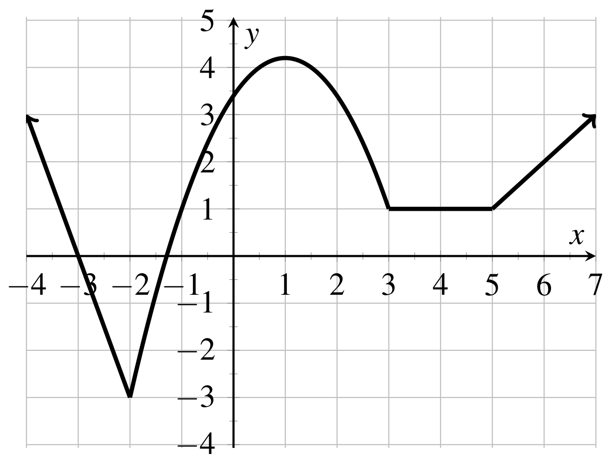
b. Use your answer in part (a) to find the equation of the tangent line to $f(x) = \frac{2}{\sqrt{x}}$ at the point $(4,1)$. **Simplify your answer.**

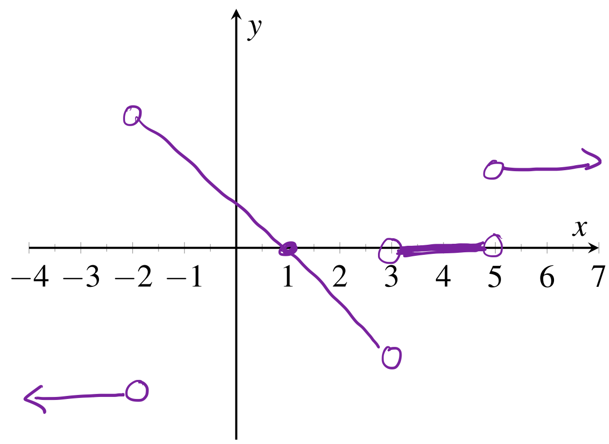
$$f'(4) = \frac{-1}{4\sqrt{4}} = \frac{-1}{8}.$$

$$y = -\frac{1}{8}(x-4) + 1 = -\frac{1}{8}x + \frac{1}{2} + 1$$

$$\boxed{y = -\frac{1}{8}x + \frac{3}{2}}$$

2. [6 points] The graph of $f(x)$ is shown below. On the **other set of axes**, sketch the graph of $f'(x)$. If there are any asymptotes, draw them with dashed lines. Use open circles to show points where the derivative is not defined, if any. (You are not given values on the y-axis; I am interested in the correct shape/holes/asymptotes of the derivative, not the specific values.)





3. [9 points] Find $f'(x)$ for each function below. You may use any method you like, and you do not need to simplify your answer.

a. $f(x) = \dfrac{8x^3 - 2\sqrt{x}}{x} - \dfrac{1}{\pi x} = 8x^2 - 2x^{-1/2} - \frac{1}{\pi}x^{-1}$ (*Hint: Make it easier, first.*)

$$f'(x) = 16x - 2\cdot\left(-\tfrac{1}{2}\right)x^{-3/2} + \frac{1}{\pi}x^{-2}$$

$$= 16x + x^{-3/2} + \frac{1}{\pi}x^{-2}$$

b. $f(x) = (3x^2 + x)\sin(x)$

$$f'(x) = \left[\frac{d}{dx}(3x^2+x)\right]\cdot\sin(x) + (3x^2+x)\left[\frac{d}{dx}\sin(x)\right]$$

$$= (6x+1)\sin(x) + (3x^2+x)\cdot\cos(x).$$

c. $f(x) = \dfrac{3\cos(x)}{2^5 + 7x}$ $\leftarrow p$, $\leftarrow g$

$p' = -3\sin(x)$ $g' = 7$

$$f'(x) = \frac{p'g - pg'}{g^2} = \frac{(-3\sin(x))\cdot(2^5+7x) - 3\cos(x)\cdot 7}{(2^5+7x)^2}$$

$$= \frac{(-3\sin(x))\cdot(2^5+7x) - 21\cos(x)}{(2^5+7x)^2}$$

Note: Simplification not required