Name: Solutions _____ / 25

Please circle your instructor's name: James Gossell Gordon Williams

There are 25 points possible on this quiz. Any outside materials are not allowed. **For full credit, show all work clearly.**

1. [15 points] Find the derivative of each function below. You do not need to simplify your answer.

a. $f(x) = \sqrt{\tan^{-1} x} = (\arctan(x))^{1/2}$

$$f'(x) = \frac{1}{2}(\arctan(x))^{-1/2} \cdot \frac{1}{1+x^2}$$

b. $g(\theta) = \sin^{-1}(\theta) - (\sin(\theta))^{-1}$

$$g'(\theta) = \frac{1}{\sqrt{1-\theta^2}} + (\sin(\theta))^{-2} \cdot \cos\theta$$

c. $h(t) = \arccos(t \ln t)$

$$h'(t) = \frac{-1}{\sqrt{1-(t\ln t)^2}} \cdot \left(\ln t + t \cdot \frac{1}{t}\right)$$

d. $j(x) = \dfrac{\arcsin(x^2)}{e^{2x}+1}$

$$j'(x) = \frac{\frac{1}{\sqrt{1-x^4}} \cdot 2x\,(e^{2x}+1) - \arcsin(x^2) \cdot 2e^{2x}}{(e^{2x}+1)^2}$$

e. $k(x) = \ln(2x^2+3x) + e^{3x^2-x} + e^{\ln \pi}$

$$k'(x) = \frac{4x+3}{2x^2+3x} + (6x-1)e^{3x^2-x}$$

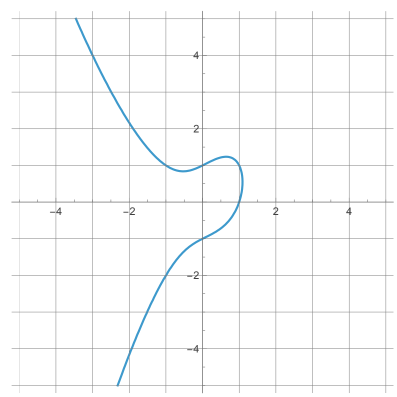
2. [4 points] Use **logarithmic differentiation** to find the derivative of the function $f(x) = \sin x^{\tan x}$.

$y = (\sin x)^{\tan x}$ $\qquad$ $\ln(y) = \tan x \cdot \ln(\sin(x))$

$\frac{1}{y} y' = \sec^2(x) \cdot \ln(\sin(x)) + \tan(x) \cdot \frac{1}{\sin(x)} \cdot \cos(x)$

$y' = (\sin(x))^{\tan(x)} \left[\sec^2(x) \ln(\sin(x)) + 1 \right]$

3. [6 points] The graph of $xy = x^3 + y^2 - 1$ is given at right.



a. Calculate $\frac{dy}{dx}$.

$y + x y' = 3x^2 + 2y y'$

$x y' - 2y y' = 3x^2 - y$

$\frac{dy}{dx} = y' = \frac{3x^2 - y}{x - 2y}$

b. Use $\frac{dy}{dx}$ to find the **equation** of the tangent line to the curve at the point $(-1, -2)$. **Simplify** your answer.

$m = \frac{3 \cdot (-1)^2 - (-2)}{-1 - 2(-2)} = \frac{3+2}{3} = \frac{5}{3}$

$y + 2 = \frac{5}{3}(x+1)$ $\quad$ or $\quad$ $y = \frac{5}{3}x + \frac{5}{3} - \frac{6}{3}$

so $\boxed{y = \frac{5}{3}x - \frac{1}{3}}$