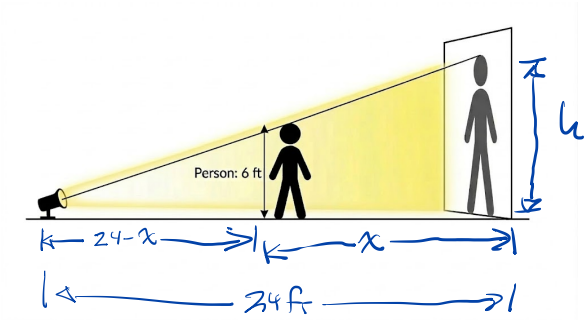
Name: Solutions _____ / **25**

Please circle your instructor's name: James Gossell Gordon Williams

There are 25 points possible on this quiz. Any outside materials are not allowed. **For full credit, show all work clearly.**

1. **[9 points]** A 6-ft-tall person walks toward a wall at a rate of 3 ft/sec. A spotlight is located on the ground 24 ft from the wall. How fast does the height of the person's shadow on the wall change when the person is 12 ft from the wall?



By similar triangles: $\frac{h}{24} = \frac{6}{24-x}$.

Know: $\frac{dx}{dt} = -3 \text{ ft/sec}$

Want: $\frac{dh}{dt}$ when $x = 12$ ft.

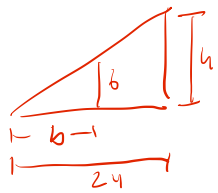
$\frac{d}{dt}\left(\frac{h}{24}\right) = \frac{d}{dt}\left(\frac{6}{24-x}\right)$ so $\frac{1}{24}\frac{dh}{dt} = 6\cdot(-1)(24-x)^{-2}\cdot(-1)\cdot\frac{dx}{dt}$

or $\frac{dh}{dt} = \frac{24\cdot 6}{(24-x)^2}\cdot\frac{dx}{dt}$. When $x = 12$ we have

$$\left.\frac{dh}{dt}\right|_{x=12} = \frac{2406 \text{ ft}^2}{12^2 \text{ ft}^2}\cdot(-3) \text{ ft/sec} = \frac{12\cdot 2\cdot 6\cdot(-3)}{12\cdot 12} \text{ ft/sec}$$

$$= -3 \text{ ft/sec}$$

Alt:



$\frac{db}{dt} = 3 \text{ ft/sec}$ $\frac{6}{b} = \frac{h}{24}$

$-6b^{-2}\cdot b' = \frac{1}{24}h'$ or $h' = \frac{-6\cdot 24\cdot b'}{b^2}$

or $b = 12$, get $h' = \frac{-6\cdot 24\cdot(3 \text{ ft/sec})}{12^2} = -3 \text{ ft/sec}$

2. [8 points] Suppose I want to estimate the value of $3e^{0.1}$.

a. Determine an appropriate $f(x)$ and a, and evaluate $L(x) = f(a) + f'(a)(x-a)$ at an appropriate point to estimate the value of $3e^{0.1}$.

$f(x) = 3e^x$. $\quad f'(x) = 3e^x \quad f'(0) = 3$.

$L(x) = 3 + 3(x-0) = 3+3x$

$3e^{0.1} \approx L(0.1) = 3.3$.

b. Write down what you would need to enter into a calculator to determine the numerical error in the linear approximation you've just obtained.

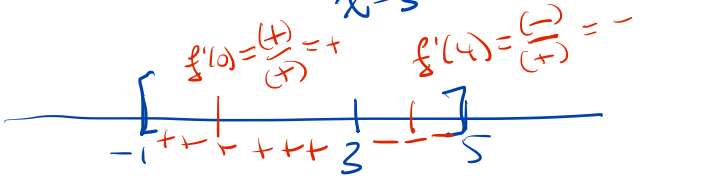
error $= e^{0.1} - 3.3$

3. [8 points] Consider the function $f(x) = \dfrac{2x}{x^2+9}$.

a. Find the critical points of $f(x)$ over the domain $[-1,5]$. Note that $f'(x) = \dfrac{18-2x^2}{(x^2+9)^2}$.

critical points when $f'(x) = 0$ or DNE.

Here $18-2x^2 = 0$ or $x^2 = 9$ or $x = \pm 3$, but only $x=3$ in domain.



b. Find the local and/or absolute extrema for the function $f(x)$ over the domain $[-1,5]$, or explain why none exist. Clearly label any you identify.

x	$f(x)$
-1	$-2/10 = -1/5$
3	$6/18 = 1/3$
5	$10/(25+9) = 10/34 = 5/17 < 5/15 = 1/3$

so: $(3, 1/3)$ abs. max

$(-1, -1/5)$ abs. min

$(5, 5/17)$ loc. min.