Name: Solutions _____ / 25

Please circle your instructor's name: James Gossell Gordon Williams

There are 25 points possible on this quiz. Any outside materials are not allowed. **For full credit, show all work clearly.**

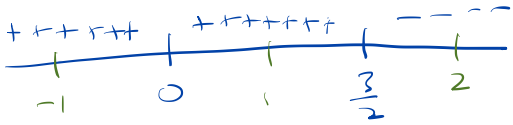
1. [12 points] The following questions concern the function $f(x) = 2x^3 - x^4$.

a. Find $f'(x)$ and identify all critical points of $f(x)$.

$f'(x) = 6x^2 - 4x^3 = x^2(6-4x)$. $f'(x) = 0$ when $x=0$ or $6-4x=0$

so critical points at $x = 0, \frac{3}{2}$.

b. Determine intervals where $f(x)$ is increasing or decreasing.



$f'(-1) > 0$ $f'(1) > 0$ $f'(2) < 0$

f is increasing on $(-\infty, 0) \cup (0, \frac{3}{2})$

f is decreasing on $(\frac{3}{2}, \infty)$

c. Identify the location (x-values) of any local maxima or minima of $f(x)$ or state that none exist.

Local maximum at $x = 3/2$.

No local minima

d. Find $f''(x)$.

$f''(x) = 12x - 12x^2 = 12x(1-x)$

e. Determine intervals where $f(x)$ is concave up and concave down.

$f''(x) = 0$ at $x = 0, 1$.

$f''(x) < 0$ for $x < 0$ $f''(x) < 0$ for $x > 1$.

$f''(x) > 0$ for $0 < x < 1$

f concave up on $(0,1)$.

f concave down on $(-\infty, 0) \cup (1, \infty)$

f. Identify the x-values of any inflection points of $f(x)$ or state that none exist.

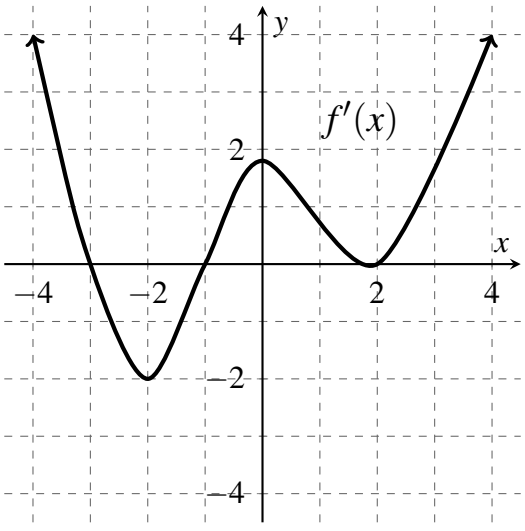
inflection points at $x = 0, 1$.

2. [8 points] Evaluate the given limits. Give the most complete answer possible. If the limit is ∞ or $-\infty$, state this. You must justify your answer algebraically. Answers without any work will not receive full credit.

a. $\lim_{x\to\infty} \dfrac{x^2-2x^4}{10x^4-x}$ $\cdot \dfrac{1/x^4}{1/x^4} = \lim_{x\to\infty} \dfrac{\frac{1}{x^2}-2}{10-\frac{1}{x^3}} = \dfrac{-2}{10} = \dfrac{-1}{5}$

b. $\lim_{x\to-\infty} \dfrac{\sqrt{2x^2}}{x-5} = \lim_{x\to-\infty} \dfrac{|x|\cdot\sqrt{2}}{x-5} \cdot \dfrac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to-\infty} \dfrac{-1\cdot\sqrt{2}}{1-5/x} = -\sqrt{2}$

3. [5 points] Below is the **graph of the derivative of f(x)**. Use this graph to answer the questions $f(x)$.



a. When is $f(x)$ increasing?

$(-\infty, 3) \cup (-1, 2) \cup (2, \infty)$

b. When is $f(x)$ decreasing?

$(-3, -1)$

c. Determine where $f(x)$ has a local maximum or a local minimum.

local max. at $x = -3$

local min. at $x = -1$.