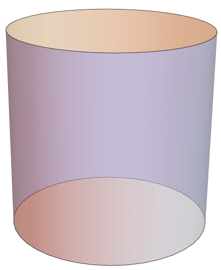
Name: Solutions ____________ / 25

Please circle your instructor's name: James Gossell Gordon Williams

There are 25 points possible on this quiz. Any outside materials are not allowed. **For full credit, show all work clearly.**

1. [10 points] Find the radius, r , and the height, h, of the closed cylinder with volume of 16π that has the least amount of surface area. The formulas for the volume, V, and the surface area, S, are given below.

$$V = \pi r^2 h, \qquad S = 2\pi rh + 2\pi r^2$$



$V = 16\pi$ so $16\pi = \pi r^2 h$. or $h = \frac{16}{r^2}$.

So $S(r) = 2\pi r \cdot \frac{16}{r^2} + 2\pi r^2 = \frac{32\pi}{r} + 2\pi r^2$.

$S'(r) = \frac{-32\pi}{r^2} + 4\pi r$.

critical points: $0 = \frac{-32\pi}{r^2} + 4\pi r \rightarrow 4\pi r = \frac{32\pi}{r^2} \rightarrow r^3 = 8 \Rightarrow r = 2$.

$S''(r) = -\frac{64\pi}{r^3} + 4\pi$. $S''(2) = \frac{-64\pi}{8} + 4\pi = -4\pi < 0$. So 2 is a local minimum on $(0,\infty)$. Unique so absolute minimum

when $r = 2$, $h = \frac{16}{2^2} = 4$.

2. [6 points] Solve the initial value problems given below.

a. Suppose $f'(x) = 3x^2 - 1$, and $f(1) = 5$. What is $f(x)$?

$f(x) = \int 3x^2 - 1\, dx = x^3 - x + C$

$f(1) = 5 = 1^3 - 1 + C$ so $C = 5$. Thus $f(x) = x^3 - x + 5$

b. Suppose $f'(x) = 3\sin(x) + e^{-x}$ and $f(0) = 0$. What is $f(x)$?

$f(x)\ \int 3\sin(x) + e^{-x}\, dx = -3\cos(x) - e^{-x} + C$.

$0 = f(0) = -3\cdot\cos(0) - e^{-0} + C = -3 - 1 + C$ so $C = 4$

$f(x) = -3\cos(x) - e^{-x} + 4$.

3. [9 points] Evaluate the following limits. **Show your work**, including appropriate use of limit notation. If you use L'Hôpital's rule, you must indicate where you are using it by writing $\overset{H}{=}$ or $\overset{L'H}{=}$ or something similar. Use ∞ or $-\infty$ where appropriate, and if the limit does not exist, write DNE and provide a justification.

a. $\lim\limits_{x\to 0} \dfrac{(1+x)^{1/3} - 1 - \frac{1}{3}x}{x^2}$ $\underset{\text{type } \frac{0}{0}}{\overset{L'H}{=}} \lim\limits_{x\to 0} \dfrac{\frac{1}{3}(1+x)^{-2/3} - \frac{1}{3}}{2x}$ $\underset{\text{type } \frac{0}{0}}{}$ $\overset{H}{=} \lim\limits_{x\to 0} \dfrac{-\frac{2}{9}(1+x)^{-5/3}}{2}$

$= -\dfrac{1}{9}$

b. $\lim\limits_{x\to \frac{\pi}{4}} (1-\cos(x))\csc(x) = \left(1-\cos\left(\frac{\pi}{4}\right)\right)\cdot \csc\left(\frac{\pi}{4}\right) = \left(1-\frac{1}{\sqrt{2}}\right)\left(\sqrt{2}\right)$

$= \dfrac{\sqrt{2}-1}{\sqrt{2}} \cdot \sqrt{2} = \sqrt{2}-1.$

c. $\lim\limits_{x\to\infty} \left(1+\dfrac{3}{x}\right)^x$

type 1^∞

$\lim\limits_{x\to\infty} x \ln\left(1+\frac{3}{x}\right)$ type $\infty \cdot 0$ $= \lim\limits_{x\to\infty} \dfrac{\ln(1+3/x)}{1/x}$ type $\frac{0}{0}$

$\overset{H}{=} \lim\limits_{x\to\infty} \dfrac{\frac{1}{1+3/x}\cdot\left(\frac{-3}{x^2}\right)}{-1/x^2} = \lim\limits_{x\to\infty} \dfrac{3}{1+\underbrace{\frac{3}{x}}_{0}} = 3.$

So $\lim\limits_{x\to\infty} \left(1+\frac{3}{x}\right)^x = e^3.$