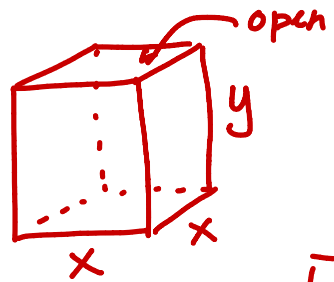
Name: ____

All of the problems below are homework problems. The goal in Recitation is to get all of these problems **set up.** For each problem you want to (a) draw and label a picture and pick notation, (b) identify what you want to maximize or minimize, (c) write the thing from part (b) as a function of **one** variable.

Problem A You are going to construct an open-topped box with a square base. You need the box to have a volume of $10m^3$. The material for the base costs $\$5/m^2$ and the material for the sides costs $\$2/m^2$. Find the dimensions of the box that minimize the cost.



$V = x^2 y = 10\ m^3 \longmapsto$ Solve for y: $y = 10x^{-2}$

$C = \$5 \cdot x^2 + \$2 \cdot 4 \cdot xy = 5x^2 + 8xy$

$C(x) = 5x^2 + 8x\left(\frac{10}{x^2}\right) = \underline{\underline{5x^2 + 80x^{-1}}}$

$\boxed{\text{goal: minimize } C(x)}$

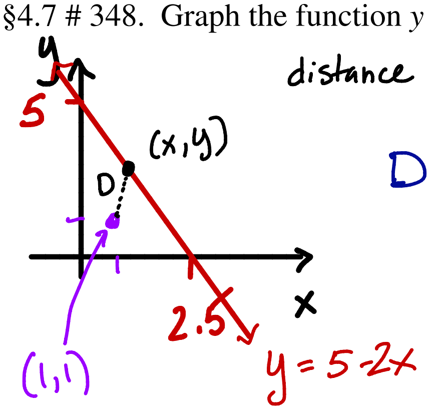
§4.7 # 332. A pizzeria sells pizzas for a revenue of $R(x) = ax$ and costs $C(x) = b + cx + dx^2$, with x represents the number of pizzas and R and C are given in dollars. Suppose that $a, b,$ and d are all positive.
Find the profit function for the number of pizzas. How many pizzas will maximize the profit?
Find the function for profit per pizza. How many pizzas will maximize the profit per pizza?

Profit: $P(x) = R(x) - C(x) = ax - (b + cx + dx^2)$
$= ax - b - cx - dx^2$

Profit per pizza: $Q(x) = \frac{P(x)}{x} = (a-c) - bx^{-1} - dx$

goal: maximize $P(x)$ and $Q(x)$.

§4.7 # 348. Graph the function $y = 5 - 2x$. Where is the line $y = 5 - 2x$ closest to the point $(1, 1)$.



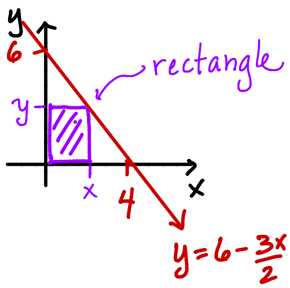
distance $= d = \sqrt{(x-1)^2 + (y-1)^2}$ ← too hard! Square both sides

$D = (x-1)^2 + (y-1)^2 = (x-1)^2 + (5-2x-1)^2$
$= (x-1)^2 + (4-2x)^2$

minimize D.

§4.7 # 343. Find the area of the largest rectangle that fits into the triangle with sides $x = 0$, $y = 0$, and $6 \cdot \left(\frac{x}{4} + \frac{y}{6} = 1.\right)$

$\frac{3x}{4} + y = 6$ or $y = 6 - \frac{3x}{2}$

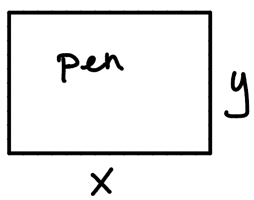


$A = xy = x\left(6 - \frac{3}{2}x\right)$

$A(x) = 6x - \frac{3}{2}x^2$

goal: maximize A

§4.7 # 319. You have 400 ft of fencing to construct a rectangular pen for cattle. What are the dimension of the pen that maximize the area?



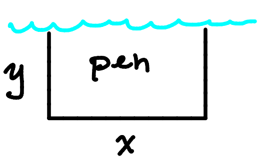
$A = xy$

perimeter $= 400 = 2x + 2y$; So $y = \frac{400 - 2x}{2} = 200 - x$

$A(x) = x(200 - x) = 200 - x^2$

maximize area A.

§4.7 # 320. You have 800 ft of fencing to make a pen for hogs. If you have a river on one sider of your property, what is the dimension of the regangular pen that maximizes the area?



$2y + x = 800$; So $x = 800 - 2y$

$A = xy$

$A(y) = (800 - 2y)y = 800y - 2y^2$

maximize area.

§4.7 # 324. A patient's pulse measures 70 bpm, 80 bpm, and 120 bpm. To determine an accurate measurement ofpulse, the doctor want to know what value minimizes the expression $(x-70)^2 + (x-80)^2 + (x-120)^2$. Why would a doctor want to minimize this? What value minimizes it?

$f(x) = (x-70)^2 + (x-80)^2 + (x-120)^2$

minimize $f(x)$

Why? The x that minimizes f is closest to all three measurements simultaneously.