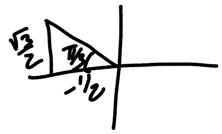


1. Review Topic

   (a) Write an equation of the line tangent to the graph of $f(x) = \frac{\sin(x)}{2}$ at $x = 2\pi/3$.

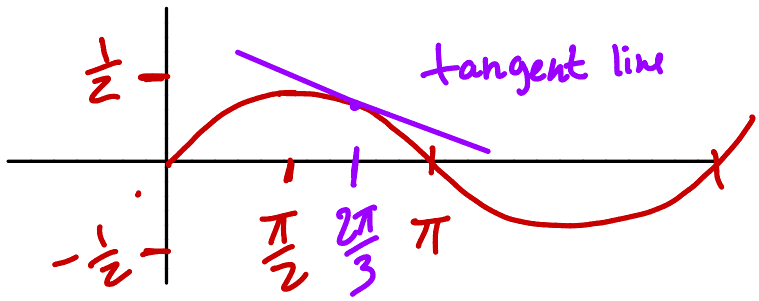
   $f\left(\frac{2\pi}{3}\right) = \frac{1}{2}\sin\left(\frac{2\pi}{3}\right) = \frac{1}{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

   $f'(x) = \frac{1}{2}\cos(x)$, $f'\left(\frac{2\pi}{3}\right) = \frac{1}{2}\cos\left(\frac{2\pi}{3}\right) = \frac{1}{2}\cdot\frac{-1}{2} = \frac{-1}{4}$

   line: $y - \frac{\sqrt{3}}{4} = \frac{-1}{4}\left(x - \frac{2\pi}{3}\right)$

   $y = \frac{\sqrt{3}}{4} - \frac{1}{4}\left(x - \frac{2\pi}{3}\right)$

   (b) On the same set of axes, make a sketch of $f(x)$ and the tangent line from part (a).



   $y = \frac{1}{2}\sin(x)$

   (c) Does your answer in part (a) seem plausible given your sketch in part (b)? Explain.

   Yes. The slope is negative, not too steep.

2. Weekly Exponential / Logarithm Practice

   (a) Determine whether the equalities below are true or false. Explain how you can remember which is which.

$e^x e^{2x} = e^{3x}$	$e^x e^{2x} = e^{2x^2}$	$e^x + e^{2x} = e^{x+2x}$
T	F	F
$e^a e^b = e^{a+b}$	pick $x=1$ $e \cdot e^2 = e^3$ e^2	try $x=1$ again $e^1 + e^2 \neq e^3 \approx 27$ $\approx 3+9$
$(e^{x+1})^2 = e^{2(x+1)}$	$(e^{x+1})^2 = e^{((x+1)^2)}$	$(e^{x+1})^2 = e^{2x}e^2$
T	F	T
$(e^a)^b = e^{ab}$	Pick $x=3$. $(e^4)^2 = e^4 \cdot e^4 = e^8$ $e^{((4)^2)} = e^{16}$	$(e^{x+1})(e^{x+1})$ $= e^{x+1+x+1} = e^{2x} \cdot e^2$

(b) Logarithmic expressions can be rewritten as an exponential expressions. (See example below.)

$$\boxed{\log_{10} A = B} \text{ is equivalent to } \boxed{10^B = A}$$

i. Rewrite each logarithmic expressions as an exponential expression.

$\boxed{\log_2 A = B}$ is equivalent to $2^B = A$

$\boxed{\ln A = B}$ is equivalent to $e^B = A$

ii. In each expression below, find y by rewriting the logarithmic expression as an exponential expression.

$\log_2 \frac{1}{32} = y$

$2^y = \frac{1}{32} = 2^{-5}$

$y = -5$

$\ln(e^{4.76}) = y$

$e^y = e^{4.76}$

$y = 4.76$

$\log_{10} y = 1$

$10^1 = y$

$y = 10$

In the following problems, you will practice the simplification skills needed to efficiently find derivatives in Section 3.3 and 3.4.

3. Write each expression using a single exponent with x in the numerator, if possible.

(a) $x^a x^b = x^{a+b}$

(b) $x^a + x^b$ (nothing to do)

(c) $(x^a)^b$ x^{ab}

(d) $\frac{x^a}{x^b}$ x^{a-b}

(e) (a and b are integers) $\sqrt[a]{x^b} = x^{b/a}$

(f) (a and b are integers) $(\sqrt[a]{x})^b = x^{b/a}$

(g) $\frac{1}{x^a} = x^{-a}$

4. Write each expression below such that x has a single exponent and so that x is in the numerator.

(a) $x^5 x^{-1/2} = x^{5-\frac{1}{2}} = x^{9/2}$

(b) $(x^5)^{-1/2} = x^{-5/2}$

(c) $\frac{3x^{-5}}{x^4} = 3x^{-9}$

(d) $\frac{x^{-5}}{3x^{-3}} = \frac{1}{3}x^{-2}$

(e) $\frac{2}{\sqrt[3]{x^2}} = 2x^{-2/3}$

(f) $\frac{x}{\sqrt{x}} = x^{1/2}$

5. (3.3#110)Write this expression so that fractions are not necessary.

$x^3 - \frac{2}{\sqrt{x}} + \frac{1}{x}$ $= x^3 - 2x^{-1/2} + x^{-1}$

6. (3.3#113)Expand and simplify this expression so that no x's are in the denominator.

$x^3\left(\frac{3}{x} - \frac{1}{5x^3} + \frac{2}{x^4}\right)$ $= 3x^2 - \frac{1}{5} + 2x^{-1}$

7. (3.3#115) Write this expression so that fractions are not necessary.

$\frac{3-x^2+x^3}{x^2}$ $= 3x^{-2} - 1 + x$

8. (Write this expression so that fractions are not necessary) $(\frac{x^2}{x^4+1})^4$ $= x^8(x^4+1)^{-4}$

9. Identify which of the following equalities are true and which are false. For the ones that are false, give some reason. For the ones that are true, do the algebraic steps that demonstrate this.

   (a) $\frac{1}{\sqrt{3x}} = \frac{x^{-1/2}}{\sqrt{3}}$ T $\frac{1}{\sqrt{3x}} = \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{3}} x^{-1/2}$

   (b) $\sqrt{4+x} = 2 + \sqrt{x}$ F. Try $x = -3$.

   (c) $\sqrt{27x^5} = 3\sqrt{3}x^{5/2}$ T $\sqrt{27x^5} = \sqrt{27}\sqrt{x^5} = 3\sqrt{3}\, x^{5/2}$

   (d) $\sqrt{x^2 - 25} = x - 5$ F. Try $x=0$.

   (e) $\frac{1}{2x} = 2x^{-1}$ F $\frac{1}{2x} = \frac{1}{2}x^{-1}$ or $\frac{1}{2x} = (2x)^{-1}$

Manipulating Fractions

10. Identify which of the following equalities are true and which are false. For the ones that are false, give some reason. F~~or the ones that are true, do the algebraic steps that demonstrate this.~~

    (a) $\frac{a+b}{c+d} = \frac{a}{c+d} + \frac{b}{c+d}$ T

    (b) $\frac{a+b}{c+d} = \frac{a+b}{c} + \frac{a+b}{d}$ F

    (c) $\frac{a+b}{c+d} = (a+b)(c+d)^{-1}$ T

    (d) (3.3 #115) $\frac{x+x^2}{2x} = \frac{1+x^2}{2}$ T

    (e) (3.3 #114) $\frac{t^2-2t-\pi}{8} = \frac{1}{8}t^2 - \frac{1}{4}t - \frac{\pi}{8}$ T

    (f) (3.3 #115) $\frac{1+3x-2x^3}{2x^3} = \frac{1}{2}x^{-3} + \frac{3x^{-2}}{2} - 1$ T

    (g) $\frac{x}{x-1} = x(x^{-1} - 1) = 1 - x$ F