RECITATION: WEEK 6

This worksheet is a refresher on inverse functions which is important to understanding Section 3.7.

$\boxed{\text{Exponential and Logarithm Review}}$

1. For each expression below, write its alternate form (or algebraic rule) or state that there is none. The first two have been done for you. Note that for each rule and non-rule, you want to ask, "*How* do I know this and how will I *remember* this?"

   (a) $(e^a)^b = \underline{e^{ab}}$

   (b) $e^a + e^b =$ no obvious rule
   Though you could try factoring out:
   $e^a(1 + e^{b-a})$

   (c) $e^x e^a = e^{x+a}$

   (d) $3^x 9^x = 3^x \cdot 3^{2x} = 3^{3x}$

   (e) $\ln(ab) = \ln(a) + \ln(b)$

   (f) $\ln(a+b) =$ no rule

   (g) $\ln(a^b) = b\ln(a)$

   (h) $\ln(\frac{a}{b}) = \ln(a) - \ln(b)$

   (i) $\ln(2e + e^2) = \ln(e(2+e)) = \ln e + \ln(2+e)$
   $= 1 + \ln(2+e)$

   (j) $\ln(1) = 0$

   (k) $\ln(0) =$ undefined

   (l) $\ln(e) = 1$

   (m) $\log_{10}(100\sqrt[3]{x}) = \log 10^2 + \log x^{1/3} = 2 + \frac{1}{3}\log x$

   (n) $3\log_{10}(x+1) - \log_{10}(2) = \log_{10}\left(\frac{(x+1)^3}{2}\right)$

   (o) $e^{5\ln(x)} = x^5$

   (p) $\ln(4e^x + 1) =$ no rule

   (q) $\ln(3e^{4x}) = \ln 3 + 4x$

$\boxed{\text{Inverse Function Review}}$

2. In your own words/pictures/examples, explain what it means for the functions $f(x)$ and $g(x)$ to be **inverses** of each other. Think of as many different ways to explain this as possible.

- f and g "undo" each other
- examples: $f(x) = x^3$, $g(x) = x^{1/3}$
  $f(x) = 2x$, $g(x) = x/2$
- $f(g(x)) = x$ and $g(f(x)) = x$
- graphs are a reflection about $y = x$

- Given $y = f(x)$. We know it's inverse is $x = f(y)$.
  Example: $y = e^x$
  $x = e^y$ is $y = \ln x$.
- black box cartoon of functions:
  if $a \to \boxed{f(x)} \to b$, then
  $b \to \boxed{f^{-1}(x)} \to a$.

3. If $f(x) = \frac{1}{x-2}$, find $f^{-1}(x)$. Sketch f and f^{-1} on the same set of axes. Check that your formula for f^{-1} is correct using two methods: (1) use a particular value, say $x = 4$ and (2) by composition, say $f(f^{-1}(x))$

find f^{-1}

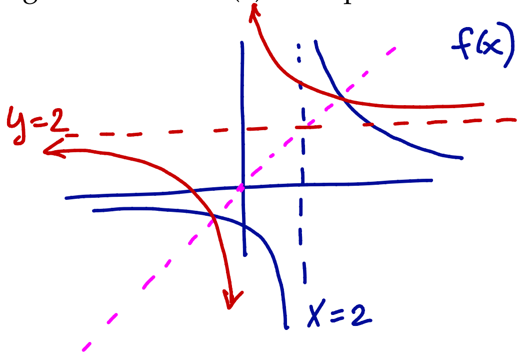
$y = \frac{1}{x-2}$

$x = \frac{1}{y-2}$

$y - 2 = \frac{1}{x}$

$y = \frac{1}{x} + 2$

$f^{-1}(x) = \frac{1}{x} + 2$



check using composition:

$f(f^{-1}(x)) = f\left(\frac{1}{x} + 2\right)$

$= \frac{1}{\left(\frac{1}{x} + 2\right) - 2}$

$= \frac{1}{\frac{1}{x}} = x$ ✓

check!

check correctness using $x=4$:

$f(4) = \frac{1}{4-2} = \frac{1}{2}$

$f^{-1}\left(\frac{1}{2}\right) = \frac{1}{\frac{1}{2}} + 2 = 2+2$
$= 4$

check!

4. Several points on the graph of $y = f(x)$ are listed in the table below. Use this table to answer the questions below, *if possible.*

x	-3	-2	-1	0	1	2
$f(x)$	8	4	2	1	0.5	0.25

(a) $f^{-1}(1) =$ 0

(b) $f^{-1}(2) =$ -1

(c) $f^{-1}(4) =$ -2

(d) $f^{-1}(0) =$ don't know. No zero here!

(e) domain of $f^{-1}(x)$? 0.25 to 8 ?

(f) range of $f^{-1}(x)$? -3 to 2 ?

best guesses given the limited info in table...

5. **The notation for inverse functions is confusing!!** In each case below, explain why the two functions (i) and (ii) are different.

(a) $f(x) = x^3$: (i) $f^{-1}(x)$ and (ii) $(f(x))^{-1}$

$f^{-1}(x) = x^{1/3}, \quad (f(x))^{-1} = \frac{1}{x^3} = x^{-3}$

(b) (i) $g(x) = \sin^{-1}(x)$ and (ii) $h(x) = (\sin(x))^{-1}$

$g(x) = \arcsin(x) \qquad h(x) = \frac{1}{\sin(x)} = \csc(x)$

6. Explain why the -1's (or -3's mean different things in the expressions below and explain **how you can tell the difference**:

x^{-1}	$f^{-1}(x)$	$2x^{-3}$	$\tan^{-3}(x)$	$\tan^{-1}(x)$	$(\tan(x))^{-1}$	$(2x)^{-3}$
$= \frac{1}{x}$	inverse of $f(x)$	$\frac{2}{x^3}$	$\frac{1}{(\tan x)^3}$	$\arctan(x)$	$\frac{1}{\tan x} = \cot(x)$	$= \frac{1}{8x^3}$

7. If $f(x) = e^x$, what is $f^{-1}(x)$? Write out the two identities obtained from $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. Use your calculator to confirm these identities using $x = 2$.

$f^{-1}(x) = \ln(x)$

$f(f^{-1}(x)) = \boxed{e^{\ln(x)} = x}$

$f^{-1}(f(x)) = \boxed{\ln(e^x) = x}$

$e^{\ln(2)} = e^{0.6931418} = 1.99999999888 \approx 2$

check!

8. If $f(x) = \sin(x)$, what is $f^{-1}(x)$? Write out the two identities obtained from $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. Use your calculator to confirm these identities using $x = 120°$

$f^{-1}(x) = \arcsin(x)$

$f(f^{-1}(x)) = \boxed{\sin(\arcsin(x)) = x}$

$f^{-1}(f(x)) = \boxed{\arcsin(\sin(x)) = x}$

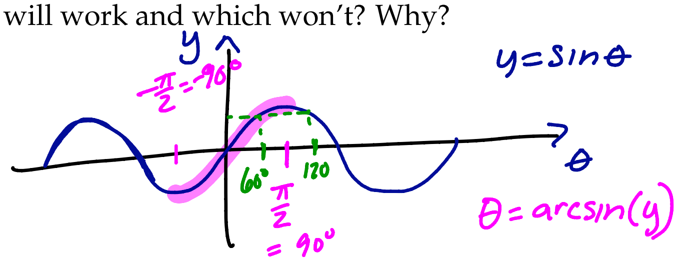
$\arcsin(\sin(120°)) = \arcsin(0.8660) = 60°$

problem... :(

9. What went wrong in the last part of #8? What x-values will work and which won't? Why?

What went wrong?

Answer: There are many different angles that give the same output from $\sin(\theta)$. Arcsine picks one.

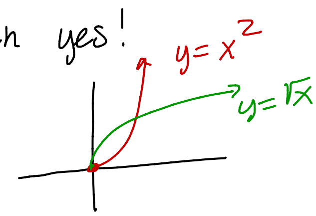


10. Are the functions $f(x) = x^2$ and $g(x) = \sqrt{x}$ inverses of each other or not? Why?

Well... kind of...

If x's put into f(x) are all nonnegative, then yes!

$f(2) = 2^2 = 4, \quad f^{-1}(4) = \sqrt{4} = 2$



If x's put into f(x) are negative, then No!

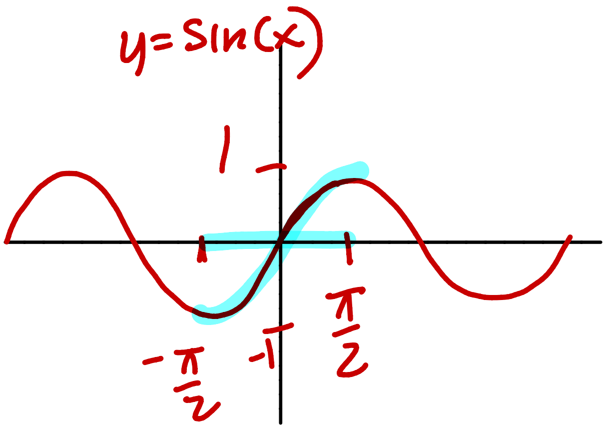
$f(-2) = (-2)^2 = 4, \quad f^{-1}(4) = \sqrt{4} = 2$

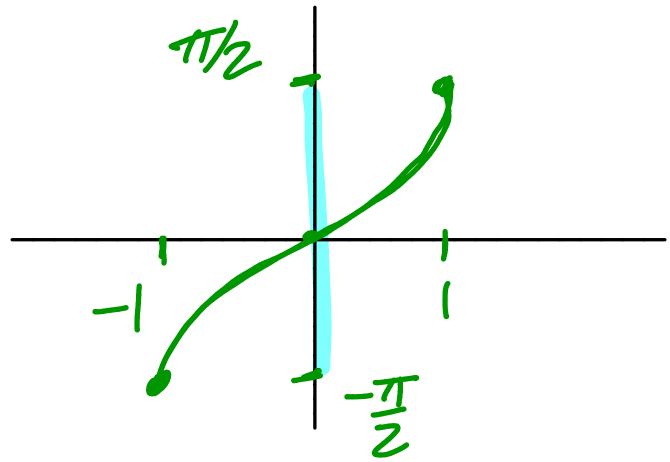
bad!

Lesson: If different x-values give the same y-value for $y = f(x)$, $f^{-1}(x)$ cannot be defined.

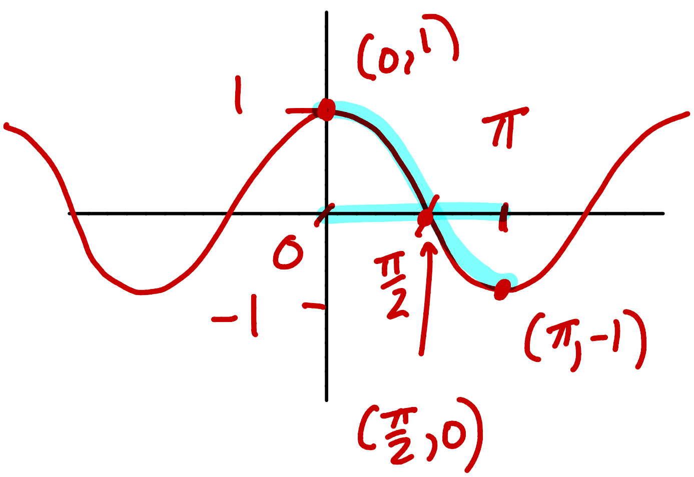
Solution: Restrict the domain of $f(x)$.

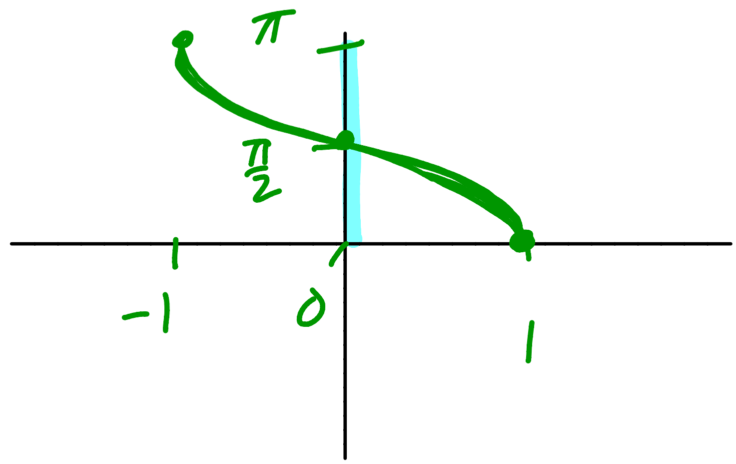
11. Graph $f(x) = \sin(x)$ and $f^{-1} = \sin^{-1}(x)$ on the ~~same~~ different set of axes.





12. Graph $f(x) = \cos(x)$ and $f^{-1} = \cos^{-1}(x)$ on the ~~same~~ different set of axes.





13. Graph $f(x) = \tan(x)$ and $f^{-1} = \tan^{-1}(x)$ on the ~~same~~ different set of axes.