## RECITATION: WEEK 7

This work sheet has three parts:

- a mini-practice derivative proficiency
- prep for 3.9 homework
- prep for 4.1 homework

### Mini-Derivative Proficiency

You have 15 minutes to write the derivatives of the six functions (three on this page and three on the next). You do not need to simplify. You do need to identify that you are taking the derivative (i.e. write $y'$ or $f'(x)$ or whatever is proper notation.)

1. $g(x) = 4x^{\pi} - e^2$

$$g'(x) = 4\pi x^{\pi-1} - 0 = 4\pi x^{\pi-1}$$

2. $f(\theta) = \theta \tan(2\theta)$

$$f'(\theta) = 1 \cdot \tan(2\theta) + \theta \cdot \sec^2(2\theta)(2)$$
$$= \tan(2\theta) + 2\theta \sec^2(2\theta)$$

3. $y = \dfrac{-3}{\sqrt{4-x^2}} = -3(4-x^2)^{-1/2}$

$$y' = -3\left(-\tfrac{1}{2}\right)(4-x^2)^{-3/2}(-2x) = \frac{-3x}{(4-x^2)^{3/2}}$$

4. $f(x) = \frac{\tan^{-1}(5x)}{10}$ $= \frac{1}{10} \tan^{-1}(5x)$

$$f'(x) = \left(\frac{1}{10}\right)\left(\frac{1}{1+(5x)^2}\right)(5) = \frac{1}{2(1+25x^2)}$$

5. $y = 2x + \sqrt{3x + \sin(4x)}$ $= 2x + (3x + \sin(4x))^{\frac{1}{2}}$

$$y' = 2 + \frac{1}{2}(3x + \sin(4x))^{-1/2}(3 + 4\cos(4x))$$

$$= 2 + \frac{3 + 4\cos(4x)}{2\sqrt{3x+\sin(4x)}}$$

6. Find $dy/dx$ for $x^3y^3 = x + y$

$$3x^2y^3 + x^3 \cdot 3y^2\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$3x^3y^2\frac{dy}{dx} - \frac{dy}{dx} = 1 - 3x^2y^3$$

$$\frac{dy}{dx}(3x^3y^2 - 1) = 1 - 3x^2y^3$$

$$\frac{dy}{dx} = \frac{1-3x^2y^3}{3x^3y^2-1}$$

1. Complete the derivative formulas below:

   (a) $\frac{d}{dx}[e^x] =$ $e^x$

   (b) $\frac{d}{dx}[10^x] =$ $\ln(10)\, 10^x$

   (c) $\frac{d}{dx}[\ln(x)] =$ $\frac{1}{x}$

   (d) $\frac{d}{dx}[\log_{10}(x)] =$ $\frac{1}{\ln(10)\, x}$

2. Complete the algebraic rules below:

   (a) $e^{a+b} =$ $e^a e^b$

   (b) $\ln(ab) =$ $\ln a + \ln b$

   (c) $\ln(a^r) =$ $r \ln(a)$

3. Find the derivative of $f(x) = e^{5x+2}$ in two ways: (a) without rewriting the function and (b) rewriting the function using the rule on exponents above.

(a) $f'(x) = 5e^{5x+2}$

(b) $f(x) = e^2 e^{5x}$

$f'(x) = e^2 \cdot e^{5x} \cdot 5 = 5e^2 e^{5x} = 5e^{5x+2}$

4. Find the derivative of $f(x) = \ln(5x^3)$ in two ways: (a) without rewriting the function and (b) rewriting the function using the rule on logarithms above. Show that your answers are the same.

(a) $f'(x) = \frac{1}{5x^3} \cdot 5 \cdot 3 \cdot x^2$

$= \frac{3}{x}$

(b) $f(x) = \ln(5x^3) = \ln 5 + 3 \ln x$

$f'(x) = 0 + 3 \cdot \frac{1}{x} = \frac{3}{x}$

5. For each expression below, take the logarithm of both sides. Then use rules of logarithms to expand the right-hand side of the expression.

   (a) $y = x(\cos(x))^x$

$\ln(y) = \ln\left(x(\cos(x))^x\right) = \ln(x) + x\ln(\cos(x))$

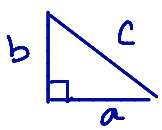
   (b) $y = \frac{x\sqrt{x^2+1}}{(3x+5)^{7/5}}$

$\ln(y) = \ln\left[\frac{x(x^2+1)^{1/2}}{(3x+5)^{7/5}}\right] = \ln(x) + \frac{1}{2}\ln(x^2+1) - \frac{7}{5}\ln(3x+5)$

4.1 Homework Help

1. You will need the following formulas. Draw and label a picture for each formula.

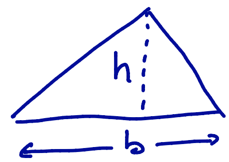
   (a) The Pythagorean Theorem

   $a^2+b^2=c^2$

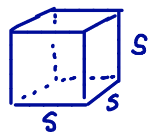
   (b) area of a triangle with height $h$ and base $b$

   $A=\frac{1}{2}bh$

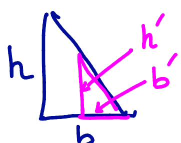
   
   

   (c) volume of a cube

   $V=s^3$

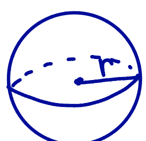
   
   

   (d) sides of similar triangles (both right triangles and isosceles triangles)

   $\frac{h}{b}=\frac{h'}{b'}$

   (e) volume and surface area of a sphere of radius $r$.

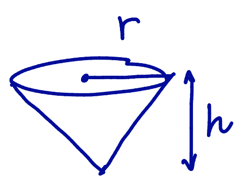
   
   

   $V=\frac{4}{3}\pi r^3$ ; $SA=4\pi r^2$

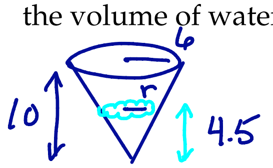
   3
   4.5
   6
   270

   (f) volume of a cone with base of radius $r$.

   $V=\frac{1}{3}\pi r^2 h$

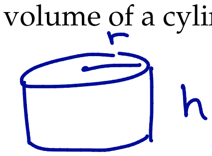
   (g) A conical water tank has a height of 10 feet and a radius of 6 feet. Assume that tank is oriented with the point down. If the height of the water in the tank is 4.5 feet high, determine the volume of water in the tank.

   $\frac{r}{6}=\frac{4.5}{10}$, So $r=\frac{6(4.5)}{10}=\frac{27}{10}=2.7ft$

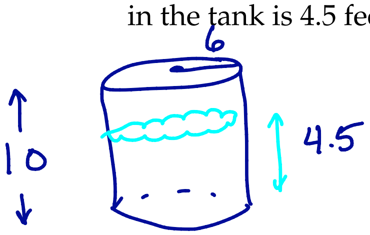
   $V=\frac{\pi(2.7)^2(4.5)}{3}$

   (h) volume of a cylinder with height $h$ and radius $r$

   $V=\pi r^2$

   (i) A cylindrical water tank has a height of 10 feet and a radius of 6 feet. If the height of the water in the tank is 4.5 feet high, determine the volume of water in the tank.

   $V=\pi\cdot 6^2(4.5)$