1. (Net Change Extra) An airplane is descending. Its rate of change of height is $r(t) = -4t + \frac{t^2}{10}$ meters per second.

   (a) If $A(t)$ is the altitude of the airplane in meters, how are $A(t)$ and $r(t)$ related?

   $A'(t) = r(t)$

   (b) What physical quantity does $\int_1^3 r(t)\,dt$ represent? The change in height of the plane in meters between second 1 and second 3.

   (c) Compute $A(3) - A(1)$.

   $-6+2=-4$

   $\frac{26}{30} = \frac{13}{15}$

   $$A(3)-A(1) = \int_1^3 r(t)dt = \int_1^3 \left(-4t + \frac{1}{10}t^2\right)dt = -2t + \frac{1}{30}t^3\Big]_1^3 = \left(-6+\frac{27}{30}\right) - \left(-2+\frac{1}{30}\right)$$

   $$= -4 + \frac{13}{15} \approx -3.1\overline{3}$$

   (d) Explain why you do not know $A(t)$ exactly.

   We aren't given the altitude of the plane at any time t and there are many functions with $r(t)$ as their derivative. We don't know which one without more information.

   (e) Explain how you can find $A(3) - A(1)$ exactly without knowing $A(t)$ exactly?

   All functions with $r(t)$ as their derivative with have the same change on the interval $[1,3]$ because... the have the same rate of change... namely $r(t)$. Another way to say it is if $F(t) = A(t) + C$, then $F(3) - F(1) = A(3) - A(1)$, so the constant doesn't matter here!

2. Fill out the blanks below:

- $\int x^n dx = \frac{x^{n+1}}{n} + C$
- $\int \sin x dx = -\cos(x) + C$
- $\int \cos x dx = \sin(x) + C$
- $\int \sec^2 x dx = \tan(x) + C$
- $\int \csc^2 x dx = -\cot(x) + C$
- $\int \sec x \tan x dx = \sec(x) + C$
- $\int \csc x \cot x dx = -\csc(x) + C$
- $\int \frac{1}{x} dx = \ln|x| + C$
- $\int e^x dx = e^x + C$
- $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin(x) + C$
- $\int \frac{1}{1+x^2} dx = \arctan(x) + C$

3. For the integral $\int \sin(x)\cos(x)\,dx$, evaluate it first using $u = \sin(x)$ then using $u = \cos(x)$. Are these really equal? Justify your answer.

① $u = \sin(x)$, $du = \cos x\,dx$

$\int u\,du = \frac{1}{2}u^2 + C = \frac{1}{2}(\sin(x))^2 + C$

② $u = \cos(x)$, $du = -\sin x\,dx$

$-\int u\,du = -\frac{1}{2}u^2 + C = -\frac{1}{2}(\cos(x))^2 + C$

Use $\sin^2 x + \cos^2 x = 1$ or $\sin^2 x = 1 - \cos^2 x$

So $\frac{1}{2}\sin^2 x + C = \frac{1}{2}(1 - \cos^2 x) + C$

$= -\frac{1}{2}\cos^2 x + \left(\frac{1}{2} + C\right)$

$= -\frac{1}{2}\cos^2 x + \tilde{C}$.

So if $C = 1$, $\tilde{C} = 1.5$ and so forth...

Lesson: Solutions to indefinite integrals can look different but still represent the same family.

4. Evaluate the integrals below.

(a) $\int \frac{1}{x^2+1}\,dx$

$= \arctan x$

(b) $\int \frac{x}{x^2+1}\,dx$

Let $u = x^2 + 1$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \frac{du}{u}$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2+1| + C$

(c) $\int \frac{x^2+1}{x}\,dx$

$= \int (x + x^{-1})\,dx$

$= \frac{1}{2}x^2 + \ln|x| + C$

(d) $\int \frac{\cos(\sqrt{x})}{\sqrt{x}}\,dx$

Let $u = x^{1/2}$
$du = \frac{1}{2}x^{-1/2}dx$
$2\,du = \frac{dx}{\sqrt{x}}$

$= \int \cos(\sqrt{x})\frac{dx}{\sqrt{x}}$

$= 2\int \cos u\,du$

$= 2\sin u + C$

$= 2\sin(\sqrt{x}) + C$

(e) $\int \frac{x^3}{\sqrt{x^2+1}}\,dx$

$u = x^2 + 1$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$
$x^2 = u - 1$

$= \int (x^2+1)^{-1/2} \cdot x^2 \cdot x\,dx$

$= \frac{1}{2}\int u^{-1/2}\cdot(u-1)\,du$

$= \frac{1}{2}\int u^{1/2} - u^{-1/2}\,du$

$= \frac{1}{2}\left(\frac{2}{3}u^{3/2} + 2u^{\frac{1}{2}}\right) + C$

$= \frac{1}{3}(x^2+1)^{3/2} + (x^2+1)^{1/2} + C$

(f) $\int \frac{x^2+1}{\sqrt{x}}\,dx = \int x^{-\frac{1}{2}}(x^2+1)\,dx$

$= \int \left(x^{\frac{3}{2}} + x^{-1/2}\right)dx$

$= \frac{2}{5}x^{5/2} - 2x^{\frac{1}{2}} + C$

* In retrospect, how do you know when to:
- just integrate
- use substitution OR
- do algebra
???