1. Give an explanation in your own words for why $x = \frac{1}{x^{-1}}$.

$$\frac{1}{x^{-1}} = \frac{1}{\frac{1}{x}} = 1 \cdot \frac{x}{1} = 1$$ ← $\boxed{\text{Your}}$ explanation is what matters!

2. Simplify $\frac{5\left(\frac{1}{x}\right)}{x^{-3}}$

$$\frac{5\left(\frac{1}{x}\right)}{x^{-3}} = \frac{5}{x} \cdot \frac{x^3}{1} = 5x^2$$

3. Evaluate the following limits being obsessive about your use of notation. Note that you must give an **algebraic** justification for your answer, possibly with the use of L'Hôpital's Rule.

(a) $\lim_{x\to\infty} \frac{\ln(x)}{\sqrt[10]{x}}$ $\stackrel{H}{=} \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{10}x^{-9/10}} = \lim_{x\to\infty} \frac{10x^{9/10}}{x} = \lim_{x\to\infty} \frac{10}{x^{1/10}} = 0$

form $\frac{0}{0}$

(b) $\lim_{x\to\infty} \frac{\sqrt{3x^2-1}}{3-x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\sqrt{3-\frac{1}{x^2}}}{\frac{3}{x}-1} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

4. What do the limits above imply about the graphs $f(x) = \frac{\ln(x)}{\sqrt[10]{x}}$ and $g(x) = \frac{\sqrt{3x^2-1}}{3-x}$?

Each has a horizonta asymptote.

$f(x)$ at $y=0$

$g(x)$ at $y=-\sqrt{3}$

5. Do either $f(x)$ or $g(x)$ have vertical asymptotes? Justify your answer.

Yes. $f(x)$ has v.a. at $x=0$. $\lim_{x\to0^+} f(x) = \lim_{x\to0^+} \frac{\overbrace{\ln x}^{\to -\infty}}{\underbrace{x^{1/10}}_{\to 0}} = -\infty.$

$g(x)$ has v.a. at $x=3$: $\lim_{x\to3^+} \frac{\overbrace{\sqrt{3x^2-1}}^{\to\sqrt{27}}}{\underbrace{3-x}_{\to 0^-}} = -\infty.$

6. Simplify $\frac{3x^2-3x+1}{2x}$ $= \frac{3}{2}x - \frac{3}{2} + \frac{1}{2}x^{-1}$

7. Determine if the following statements are True or False. **Show** your conclusion is correct. Note that the last question will ask you to revisit these problems.

T (a) $\int (3x^2+e^x)dx = x^3+e^x+C$

Check: $\frac{d}{dx}[x^3+e^x+c] = 3x^2+e^x$ ✓

T (b) $\int (3x^2+e^x)dx = x^3+e^x+18+C$

check: $\frac{d}{dx}[x^3+e^x+18+c] = 3x^2+e^x$ ✓

T (c) $\int (\ln(x)+1)dx = x\ln(x)+C$

check: $\frac{d}{dx}[x\ln(x)+c] = 1\cdot\ln(x)+x\cdot\frac{1}{x}+0 = \ln(x)+1$ ✓

F (d) $\int x\sin(x)\,dx = -\frac{1}{2}x^2\cos(x)+C$

$\frac{d}{dy}\left[-\frac{1}{2}x^2\cos(x)+c\right] = -x\cos(x)+\frac{1}{2}x^2\sin(x)$ ✗ not the same

F (e) $\int \frac{3x^2-3x+1}{2x}dx = \frac{x^3-\frac{3}{2}x^2+x}{x^2}+C$

$3x^4-2x^4 = x^4$. So $\frac{x^4}{x^4}=1$. So not equal!

$\frac{d}{dx}\left[\frac{x^3-\frac{3}{2}x^2+x}{x^2}+C\right] = \frac{x^2(3x^2-3x+1)-(x^3-\frac{3}{2}x^2+x)(2x)}{x^4}$

T

(f) $\int \sin(x)\cos(x)\,dx = \frac{1}{2}\sin^2(x) + C$

$$\frac{d}{dx}\left[\frac{1}{2}\sin^2(x)\right] = \frac{1}{2}\cdot 2\cdot \sin(x)(\cos x)$$

$$= \sin(x)\cos(x)$$

T

(g) $\int \sin(x)\cos(x)\,dx = -\frac{1}{2}\cos^2(x) + C$

$$\frac{d}{dx}\left[-\frac{1}{2}\cos^2(x)\right] = -\frac{1}{2}(2)(\cos(x))(-\sin(x))$$

$$= \sin(x)\cos(x)$$

T

(h) k is a constant, $\int (ke^x + kx)\,dx = k\int (e^x + x)\,dx$

$$ke^x + \frac{k}{2}x^2 + C$$

$$k\left(e^x + \frac{1}{2}x^2 + C\right)$$

$$= ke^x + \frac{k}{2}x^2 + \underbrace{kC}_{}$$

$$= ke^x + \frac{k}{2}x^2 + c$$

still const

F

(i) $\int (2x+3)^2\,dx = \frac{1}{3}(x^2+3x)^3 + C$

$$\frac{d}{dx}\left[\frac{1}{3}(x^2+3x)^3\right] = \frac{1}{3}\cdot 3\,(x^2+3x)^2(2x+3)$$

not the same

8. This problem asks you to go back and look at #7 above and think about what you learned from these. Before you go on, make sure you have the right answers (see the bottom of this page).

   (a) Can you always determine if an equation of the form $\int f(x)dx = F(x)+C$ is correct? If so, how? If not, why?

   Yes. Find $\frac{d}{dx}[F(x)]$. See if it's equal to $f(x)$.

   (b) Observe that 7a and 7b have the same **integrand** (namely $3x^2+e^x$) but different antiderivatives – both of which are correct. The same holds for 7f and 7g. How is this possible?

   The two differ by a constant. 7a + 7b: $C = C' - 18$.

   $\sin^2(x) + \cos^2(x) = 1$. ← So $\sin^2 x + \cos^2 x$ differ by a constant.

   (c) Equations 7d, 7e and 7i were incorrect. What do these **incorrect** expressions indicate about **WRONG** ways to evaluate indefinite integrals?

   You can't integrate a product, quotient, or composition piece by piece

   (d) You **do** have the skills to **correctly** evaluate the integrals in 4d and 7i. Do some algebra first, then evaluate the integrals.

   7d: $\int \frac{3}{2}x - \frac{3}{2} + \frac{1}{2}x^{-1}$

   $= \frac{3}{4}x^2 - \frac{3}{2}x - \frac{1}{4}x^{-2} + C$

   7i. $\int (2x+3)^2 dx = \int (4x^2+12x+9)\,dx$

   $= \frac{4}{3}x^3 + 6x^2 + 9x + C$

   (e) What rule did you learn from 7h? Write it out in a sentence.

   You can take constants OUTSIDE the "$\int$"

9. Write the equation for the top-half of the circle of radius 4 centered at $x = 10$ on the x-axis.

   $(x-10)^2 + y^2 = 16$ ← circle.

   top half: $y = \sqrt{16-(x-10)^2}$

#7. T,T,T,F,F,T,T,T,F