Below are some principles and/or integration rules you will need for the Integration Proficiency.

1. Integration Rules

   (a) $n \neq -1$, $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

   (b) $\int \frac{1}{x}\,dx = \ln|x| + C$

   (c) $\int \sin(x)\,dx = -\cos(x) + C$

   (d) $\int \cos(x)\,dx = \sin(x) + C$

   (e) $\int \sec^2(x)\,dx = \tan(x) + C$

   (f) $\int \sec(x)\tan(x)\,dx = \sec(x) + C$

   (g) $\int e^x\,dx = e^x + C$

   (h) $\int \frac{1}{1+x^2}\,dx = \arctan(x) + C$

   (i) $\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin(x) + C$

2. Each of the following attempts at integration is WRONG. Identify the error and then work the problem correctly.

   (a) $\int \frac{3x^2 - 2x}{x^{1/2}}\,dx = \frac{x^3 - x^2}{(2/3)x^{3/2}} + C$

   You can't integrate numerator + denominator separately.

   correct (Divide first)

   $$= \int \left(3x^{\frac{3}{2}} - 2x^{\frac{1}{2}}\right) dx = 3 \cdot \frac{2}{5} x^{5/2} - 2 \cdot \frac{2}{3} x^{\frac{3}{2}} + C = \frac{6}{5} x^{5/2} - \frac{4}{3} x^{\frac{3}{2}} + C$$

   (b) $\int (x-1)(2x+1)\,dx = \left(\frac{x^2}{2} - x\right)(x^2 + x) + C$

   Can't integrate a product term by term.

   Correct (multiply first)

   $$\int \left(2x^2 + x - 2x - 1\right) dx = \int \left(2x^2 - x - 1\right) dx = \frac{2}{3}x^3 - \frac{1}{2}x^2 - x + C$$

(c)

$\boxed{x \neq u}$

$$\int (x + 2x\sin(x^2+1))dx = \int (u + \sin(u))du = \frac{1}{2}u^2 - \cos(u) + C = \frac{1}{2}(x^2+1)^2 - \cos(x^2+1) + C$$

Let $u = x^2 + 1$
$du = (2x)dx$

Correct: Split the integral by the sum. Use substitution only on the "sine" part.

$$\int (x + 2x\sin(x^2+1))dx = \int x\,dx + \int 2x\sin(x^2+1)\,dx$$

Integrate

$$= \frac{1}{2}x^2 + \int 2x\sin(x^2+1)\,dx = \frac{1}{2}x^2 + \int \sin(u)\,du$$

Let $u = x^2 + 1$
$du = 2x\,dx$

$$= \frac{1}{2}x^2 - \cos(u) + C$$

$$= \frac{1}{2}x^2 - \cos(x^2+1) + C$$